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HEAT-TRANSFER TESTS OF AQUEOUS ETHYLENE GLYCOL SOLUTIONS IN AN ELECTRICALLY HEATED TUBE

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ADVANCE RESTRICTED REPORT

HEAT-TRANSFER TESTS OF AQUEOUS ETHYLENE GLYCOL SOLUTIONS IN AN ELECTRICALLY HEATED TUBE

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SUMMARY

As part of an investigation of the cooling characteristics of liquid-cooled engines, tests were conducted with an electrically heated single-tube heat exchanger to determine the heat-transfer characteristics of AN-E-2 ethylene glycol and other ethylene glycol-water mixtures for the following ranges of conditions:

Average liquid temperature, °F	100 to 250
Liquid-flow rate, pounds per second	0.17 to 2.50
Reynolds number	5,000 to 300,000
Heat flux, Btu per second per square foot	4 to 36

Similar tests were conducted with water and commercial butanol (n-butyl alcohol) for check purposes.

The results of tests conducted at an approximately constant liquid-flow rate of 0.67 pound per second (Reynolds number, 14,500 to 112,500) indicate that at an average liquid temperature of 200° F, the heat-transfer coefficients obtained using water, nominal (by volume) 30 percent-70 percent and 70 percent-30 percent glycol-water mixtures are approximately 3.8, 2.8, and 1.4 times higher, respectively, than the heat-transfer coefficients obtained using AN-E-2 ethylene glycol.

The heat-transfer coefficients of the coolants tested were satisfactorily correlated using the following equation:

$$\frac{hD}{k} = 0.048 \left(\frac{c\mu}{k}\right)^{0.4} \left(\frac{DG}{\mu}\right)^{0.73}$$

where h is heat-transfer coefficient; D is inside diameter of tube; k is thermal conductivity of liquid; c is specific heat of liquid; μ is absolute viscosity of liquid; and G is mass rate of liquid flow. In the evaluation of this equation, the physical properties used for the aqueous ethylene glycol solutions and water were those compiled by C. S. Cragoe (National Bureau of Standards) for the Coordinating Research Council.

INTRODUCTION

A satisfactory analysis of liquid-cooled engine cooling data requires a knowledge of the heat-transfer properties of the coolants used. Heat-transfer characteristics of liquids may generally be predicted from their physical properties by means of the Nusselt relation, which has been experimentally verified for a variety of liquids (reference 1, p. 181). The physical properties of ethylene glycol and ethylene glycol-water mixtures have been experimentally determined over limited temperature ranges and have been extrapolated beyond these ranges (reference 2). Few heat-transfer data, however, have been previously obtained for AN-E-2 ethylene glycol and other ethylene glycol-water mixtures; hence, the applicability of the physical properties of these coolants for a range of temperatures to the correlation of heat-transfer coefficients by established theory has not been ascertained.

As part of an investigation of the cooling characteristics of liquid-cooled engines, the tests reported herein were conducted during the winter of 1943 and the spring of 1944 at the NACA Cleveland laboratory in order to provide the heat-transfer data required for such a correlation. Forced-convection heat-transfer coefficients were determined for AN-E-2 ethylene glycol, nominal (by volume) 70 percent-30 percent and 30 percent-70 percent glycol-water mixtures for a range of average liquid temperatures, liquid-flow rates, and heat fluxes. Heat-transfer coefficients were also determined for water and commercial butanol (n-butyl alcohol) for check purposes inasmuch as heat-transfer data for these liquids are available (reference 1, pp. 180 and 181).

The tests were conducted in a modified version of the single-tube heat exchanger described in reference 3. The tube was electrically heated by the passage of current through the tube, which resulted in heat fluxes of the same order of magnitude as those prevailing in modern liquid-cooled engine cylinders.

APPARATUS

A schematic diagram of the electrically heated single-tube heat exchanger and the associated equipment used in the tests is shown in figure 1.

Heater Tube

The details of the heater-tube section, which consisted of an 18:8 stainless-steel tube with a 1/2-inch outside diameter and a 1/32-inch wall thickness, are shown in figure 2. Copper adapters were silver-soldered to each end of the tube, resulting in an effective tube length of 22.75 inches. Each adaptor was connected to a 9-inch length of 1/2-inch standard pipe (0.62 in. I.D.) and a 6-inch electric-insulating coupling of the same internal diameter (fig. 1).

Tube-wall temperatures were measured at 25 locations (fig. 2) by means of iron-constantan thermocouples (24-gage flexible-glass insulated wire) and a calibrated self-balancing indicating-type potentiometer. The thermocouples were spot-welded to the outside of the tube wall and precaution was taken that the last point of contact between the wires was at the tube surface. The tube was thermally insulated by a wrapping of flexible-glass tape, a 1-inch layer of glass wool, and a 1/4-inch layer of asbestos.

Electrical System

Power was supplied to the tube from a 208-volt alternating-current supply line through an autotransformer, a voltage regulator of the saturable-core reactor type, and a 20:1 power transformer (fig. 1). The electrical connections at the tube were made through clamp-type copper connectors. The tube was electrically insulated from the rest of the system by the nonconducting couplings.

The autotransformer and voltage-regulator unit permitted adjusting and maintaining a constant voltage and the power transformer provided large currents through the tube. A calibrated ammeter in conjunction with a 240:1 instrument current transformer was used to measure the current through the tube and a high-resistance calibrated voltmeter connected across the tube at the copper adapters was used to measure the voltage drop. The voltmeter leads were made of No. 8 solid copper wire and were maintained as short as possible in order to obviate voltmeter corrections.

Liquid System

The liquid was circulated by a centrifugal pump through a heating and cooling blending unit and then through a plate-type filter to the tube (fig. 1). From the tube the liquid flowed through a rotameter and back to the pump with a small amount of the liquid being shunted to a tank that was located above the highest point in the system. The tank provided for liquid expansion, makeup liquid, and the introduction of compressed air for conducting tests at liquid pressures above atmospheric. A bleed line from the tank was used to relieve the compressed air when tests were conducted at atmospheric pressure.

The liquid-flow rate through the tube was regulated by a throttle valve located at either end of the tube (fig. 1). The flow rate was measured with the rotameter, which had been calibrated for a range of temperatures with the various liquids used. The liquid temperature into the tube was controlled with the heating and cooling blending unit, which consisted of an electric heater, a cooler, and a mixing-valve-type temperature regulator. Liquid temperatures were measured at the entrance and the exit of the tube with single thermocouples in conjunction with the self-balancing indicating-type potentiometer.

More accurate measurements than those obtained with the single thermocouples were afforded by two thermopiles in combination with a portable potentiometer. The thermopile construction and the method of installation is illustrated in figure 3. Each of the two thermopiles consisted of four single thermocouples connected in series and distributed across the pipe diameter. The thermopiles were also connected differentially in order to measure directly the temperature rise of the liquid in flowing through the tube. The hot junction of all the liquid thermocouples was coated with an insulating varnish in order to reduce the possibility of error in the indicated temperatures resulting from electrolytic action.

Liquids and Corrosion Inhibitors

The liquids used in the tests were AN-E-2 ethylene glycol (specified on a weight basis as 94.5 percent ethylene glycol, 2.5 percent triethanolamine phosphate, and 3 percent water), water, nominal (by volume) 70-30 and 30-70 glycol-water mixtures and commercial butanol (n-butyl alcohol). The glycol concentration in AN-E-2 ethylene glycol and the more aqueous glycol mixtures was determined from the specific gravity of samples taken at intervals throughout the tests.

A corrosion inhibitor, sodium chromate, was used in preliminary tests conducted with water. This practice was discontinued before the final tests, however, because the sodium chromate was believed to be affecting the liquid thermocouple calibrations. In preliminary tests conducted with the glycol-water solutions, inconsistencies appeared in the results after very short periods of operation. These inconsistencies were probably due to fouling of the inside tube-wall surface even though the tube was thoroughly cleaned with a fine-grade steel wool before every test series. As a corrective measure, 0.2 percent by volume of NaMBT (sodium mercaptobenzothiazole) was added to the AN-E-2 ethylene glycol and the other glycol-water mixtures in the final tests. A corrosion inhibitor was not used in the tests conducted with butanol.

PRELIMINARY TESTS

Various preliminary tests were conducted in order to check the accuracy of the heater-tube instrumentation. A detailed discussion of these tests is presented in appendix A. The results of the preliminary investigation indicated that: (a) the electric currents and magnetic fields in and around the tube did not introduce any noticeable error in the tube-wall thermocouple readings; (b) end losses affected the tube-wall temperature distribution at the end sections but had little effect on the temperature distribution of the central 12 inches; and (c) the electrical resistance of the tube per inch length as calculated from the ammeter and the voltmeter readings and the length of the tube could be used for power-input computations.

FINAL TESTS AND CALCULATIONS

Final Tests

Final tests were conducted to obtain forced-convection heat-transfer coefficients for the various liquids over the following ranges of conditions:

Average liquid temperature, °F	100 to 250
Liquid-flow rate, pounds per second	0.17 to 2.50
Reynolds number	5,000 to 300,000
Heat flux, Btu per second per square foot	4 to 36

Each factor was independently varied while maintaining the other factors approximately constant. The tests were repeated at several different values of the constant factors. Most of the tests were conducted at approximately constant absolute liquid pressures of 53 to 70 pounds per square inch. In a few of the tests, however, each run was made at two different pressures in order to determine the effect of pressure on the heat-transfer coefficients. In all the tests a high enough pressure was maintained to obtain nonboiling conditions.

Calculations

The following symbols will be used in the calculations:

A_i inside area of test section of tube, (sq ft)

A_m mean area of test section of tube perpendicular to heat flow, (sq ft)

c specific heat of liquid, (Btu)/(lb)(°F)

D inside diameter of tube, (ft)

E potential drop in test section of tube, (volts)

G mass rate of liquid flow, (lb)/(sec)(sq ft)

h heat-transfer coefficient, (Btu)/(sec)(sq ft)(°F)

I tube current, (amperes)

k thermal conductivity of liquid, (Btu)/(sec)(sq ft)(°F/ft)

k_s thermal conductivity of 18:8 stainless steel, (Btu)/(sec)(sq ft)(°F/ft)

m, n exponents, experimentally determined

p absolute liquid pressure, (lb)/(sq in.)

q rate of heat input to test section of tube, (Btu)/(sec)

q' rate of heat input to entire tube length, (Btu)/(sec)

q_r rate of heat rejected to liquid, (Btu)/(sec)

R electrical resistance of test section of tube, (ohms)

r electrical resistance of tube per inch length, (ohms)/(in.)

t average liquid temperature, (°F)

t_i average inside-wall temperature of test section of tube, (°F)

t_o average outside-wall temperature of test section of tube, (°F)

t_s temperature of outside-surface of tube insulation, (°F)

Δt temperature rise of liquid in flowing through tube, (°F)

W liquid-flow rate, (lb)/(sec)

x thickness of tube wall, (ft)

y_i inside radius of tube, (ft)

y_o outside radius of tube, (ft)

α dimensionless constant

β constant, 0.000948, (Btu/sec)/(watt)

θ time, (sec)

μ absolute viscosity of liquid, (lb)/(ft)(sec)

ρ resistivity of tube, (ohms)(sq ft)/(ft)

$c\mu/k$ Prandtl number

DG/μ Reynolds number

hD/k Nusselt number

h/cG Stanton number

Average temperatures. - Average liquid temperatures t were taken as the arithmetic mean of the liquid-bulk temperatures measured at the entrance and exit ends of the tube. Average outside-tube-wall temperatures t_o were taken as the arithmetic average of the temperatures indicated by the 13 thermocouples spot-welded on the central 12 inches of the tube; that is, the test section of the tube was considered to consist of the central 12 inches in order to reduce the possibility of introducing errors in the final results owing to the effect of end losses on the tube-wall temperature distribution at the end sections. (See appendix A.) Average inside-tube-wall temperatures t_i were calculated using the following relation, which is derived in appendix B:

$$q = \frac{k_s A_m}{x} \frac{(t_o - t_i)}{0.5}$$

where A_m is equal to 0.123 square foot and k_s is obtained from figure 4, prepared from references 4 and 5, at the value of the average outside-tube-wall temperature t_o.

Power input and heat-transfer coefficients. - The power input to the tube q was calculated using the I^2R law where the total electrical resistance R is equal to the product of the resistance of the tube per inch length r and the length of the test section considered (12 in.). Figure 5 shows r as a function of temperature as determined in the check tests. Values of r were obtained at the value of the average outside-tube-wall temperature t_o.

Heat-transfer coefficients h were calculated as follows:

$$h = \frac{q}{A_i (t_i - t)}$$

where A_i is equal to 0.115 square foot.

Heat rejections and physical properties. - The total heat rejected to the liquid based on the full-length tube was calculated as follows:

$$q_r = Wc \, \Delta t$$

where Δt, the temperature rise of the liquid, was obtained from the differentially connected thermopiles.

The specific heat c, the thermal conductivity k, and the absolute viscosity μ of the liquids were determined at the value of the average liquid temperature t. Figures 6 and 7 (data from reference 2) and figure 8 (data from references 6, 7, and 8) show the physical property values of water, aqueous ethylene glycol solutions, and butanol, respectively, as a function of temperature.

The physical properties of the glycol-water mixtures were evaluated by assuming that the corrosion inhibitors in the solutions were approximately equal to an equivalent amount of ethylene glycol.

For example, the properties of AN-E-2 ethylene glycol were evaluated as for a nominal (by volume) 97-3 glycol-water mixture; hence, corrections were not made for the small effects of the corrosion inhibitors on the individual physical-property values. The errors introduced in the final results by making this assumption were relatively small.

RESULTS AND DISCUSSION

Summary of Data

A summary of data and results for all of the tests except preliminary and check tests is presented in table I. The values presented for the heat rejected to the liquid represents the total heat rejected on the basis of the full-length tube (22.75 in.). The total heat rejected to the liquids is usually lower than the total electrical heat input. The maximum deviation is less than 10 percent in most cases. The heat loss through the thermal insulation on the tube was estimated to be less than 1 percent of the heat input and the remaining portion of the total heat loss is attributed to end losses through the copper adapters and busses. At the central 12-inch test section, however, heat-input measurements should be accurate measures of heat transfer inasmuch as the effect of end loss on this portion of the tube is negligible. (See appendix A.)

Individual Heat-Transfer Coefficients

The variation of the heat-transfer coefficient with rate of heat input is shown in figure 9 from the results of tests conducted with water at an average liquid temperature of approximately 150° F, at a liquid pressure of 65 pounds per square inch absolute, and at a liquid-flow rate of 0.20 pound per second. The heat-transfer coefficients remained approximately constant for variations in the power supplied to the tube. This constant relation is, in effect, a precision check of the entire setup inasmuch as a constant liquid-flow rate and average liquid temperature (hence physical properties) predicates constant heat-transfer coefficients. (See equation (1).)

The increase of heat-transfer coefficients with average liquid temperature for water and each of the glycol-water mixtures tested is shown in figure 10. The data were obtained at approximately constant liquid-flow rates of 0.67 pound per second and different constant heat inputs. In the tests conducted with the glycol-water mixtures, a constant liquid pressure of 68 pounds per square inch absolute was maintained, whereas in the tests conducted with water each run was made at liquid pressures of 15 and 68 pounds per square inch absolute. In the water data, no appreciable effect of pressure was found. The change of the heat-transfer coefficients per degree Fahrenheit change in average liquid temperature and the value of the heat-transfer coefficients at 150° F and 200° F as obtained from figure 10 are listed in the following table:

Coolant glycol-water (percent by volume)	Slope	Heat-transfer coefficient, h (Btu)/(sec)(sq ft)(°F)	
		Average liquid temperature (°F)	
		150	200
0-100	0.16	0.75	0.83
30-70	.18	.52	.62
70-30	.13	.24	.30
97-3	.12	.16	.22

The advantage in cooling performance of water and the more aqueous glycol solutions over AN-E-2 ethylene glycol is shown in the previous table. At an average liquid temperature of 200° F, the heat-transfer coefficients obtained using water, nominal (by volume) 30-70 and 70-30, glycol-water mixtures are approximately 3.8, 2.8, and 1.4 times higher, respectively, than the heat-transfer coefficients obtained using AN-E-2 ethylene glycol.

Correlation of Heat-Transfer Coefficients

The heat-transfer coefficients were correlated using the familiar Nusselt relation (reference 1, p. 164):

$$\frac{hD}{k} = \alpha \left(\frac{c\mu}{k}\right)^n \left(\frac{DG}{\mu}\right)^m \tag{1}$$

In order to determine the exponent of the Prandtl number in equation (1) two plots were made. The first plot (fig. 11(a)) shows the Nusselt number plotted on logarithmic coordinates against the Reynolds number as obtained from variable liquid-flow-rate tests conducted with water and the glycol-water solutions at various constant values of average liquid temperature (hence Prandtl number), power input, and liquid pressure. A family of approximately parallel lines was obtained.

From figure 11(a) at a value of the Reynolds number equal to 50,000, the values of the Nusselt number are cross-plotted on logarithmic coordinates against the Prandtl number in figure 11(b). The slope of the resulting line is approximately 0.4. This value, which is equal to the exponent of the Prandtl number (equation (1)), is in agreement with that found by investigators using various other liquids (reference 1, p. 167).

The first correlation plot of the heat-transfer coefficients presented in table I is shown in figure 12 in a logarithmic plot of $\frac{hD}{k}\Big/\left(\frac{c\mu}{k}\right)^{0.4}$ against DG/μ. The results for AN-E-2 ethylene glycol and the more aqueous glycol mixtures correlate well with the water and the butanol data. The value of the slope of the resulting line through the data is approximately 0.73. Although 0.8 is generally recommended as the exponent of the Reynolds number in equation (1), investigators using other liquids have found values of the exponent from about 0.7 to 0.8 (reference 1, pp. 178-181). The equation of the line through the data is as follows:

$$\frac{hD}{k} = 0.048 \left(\frac{c\mu}{k}\right)^{0.4} \left(\frac{DG}{\mu}\right)^{0.73} \qquad (2)$$

The average scatter from this equation is approximately ±10 percent with a few of the points, especially those below a Reynolds number of 10,000, deviating slightly more than 10 percent.

The second correlation plot of the heat-transfer coefficients involving the Stanton number is shown in figure 13 in a logarithmic plot of $\left(\frac{h}{cG}\right)\left(\frac{c\mu}{k}\right)^{0.6}$ against DG/μ. This second method of correlating forced-convection heat-transfer data has the advantage of illustrating better than the first correlation plot trends in the neighborhood of the transition region. The effect of the transition is shown in figure 13 by the curvature of the data below a Reynolds number of approximately 10,000. The equation of the line for the data of Reynolds numbers greater than 10,000, which corresponds to equation (2), is:

$$\left(\frac{h}{cG}\right)\left(\frac{c\mu}{k}\right)^{0.6} = 0.048 \left(\frac{DG}{\mu}\right)^{-0.27}$$

The average scatter from this equation is approximately ±10 percent with slightly larger deviations for some of the points.

SUMMARY OF RESULTS

The results of heat-transfer tests conducted with AN-E-2 ethylene glycol, nominal (by volume) 70 percent-30 percent and 30 percent-70 percent glycol-water solutions, water, and commercial butanol under turbulent flow conditions in an electrically heated tube showed that:

1. At a liquid-flow rate of 0.67 pound per second (Reynolds number, 14,500 to 112,500) and at an average liquid temperature of 200° F, the heat-transfer coefficients obtained using water, nominal (by volume) 30 percent-70 percent and 70 percent-30 percent glycol-water mixtures are approximately 3.8, 2.8, and 1.4 times higher, respectively, than the heat-transfer coefficients obtained using AN-E-2 ethylene glycol.

2. The heat-transfer coefficients of the coolants tested were satisfactorily correlated using the following equation:

$$\frac{hD}{k} = 0.048 \left(\frac{c\mu}{k}\right)^{0.4} \left(\frac{DG}{\mu}\right)^{0.73}$$

where h is heat-transfer coefficient; D is inside diameter of tube; k is thermal conductivity of liquid; c is specific heat of liquid; μ is absolute viscosity of liquid; and G is mass rate of liquid flow. In the evaluation of this equation, the physical properties used for the aqueous ethylene glycol solutions and water were those compiled by C. S. Cragoe (National Bureau of Standards) for the Coordinating Research Council.

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APPENDIX A

PRELIMINARY TESTS

Various preliminary tests were conducted in order to check the accuracy of the heater-tube instrumentation.

Validity of measurements of outside-tube-wall temperatures. - In view of the possible existence of errors in tube-wall-temperature readings because of the electric currents and the magnetic fields in and around the electrically heated tube, cooling-rate tests were conducted to check the validity of the tube-wall temperatures obtained. These tests were conducted by supplying a small amount of power to the tube devoid of any liquid until some predetermined temperature was reached. At that point the power supply was cut off and the indications of the thermocouples were recorded at intervals of 5 seconds for approximately 1 minute so that by extrapolation to zero time, the initial temperature could be obtained.

The results of these tests are shown in figure 14 in a semilogarithmic plot of $t_o - t_s$ against θ. Linear relations were obtained from the results for the central thermocouples with slight deviations from linear relations obtained from the results for the end thermocouples. In an ideal case, straight lines would have been obtained for all of the results. The extrapolated temperatures at zero time, however, did check all of the initial tube-wall temperatures that were recorded when power was being supplied to the tube, thus indicating no appreciable error in the tube-wall-temperature readings.

Tube-wall temperature distribution. - In order to obtain an indication of: (a) the temperature distribution along the length of the tube and (b) the effect of end losses on the tube-wall temperatures, a test was conducted while the tube was dry wherein the heat input was set equal to the heat losses. After equilibrium was maintained for approximately 1/2 hour, all the thermocouple readings were recorded. The results of this test are shown in figure 15 where each tube-wall temperature is plotted with respect to the corresponding thermocouple location. The thermocouples located 9 and 10 inches from the entrance of the tube were inoperative when this test was conducted. The temperatures indicated by the other thermocouples located on the central 12 inches of the tube agreed within approximately $\pm 8^{\circ}$ F but on both sides of this central section the temperatures decreased rapidly. The resulting temperature distribution is, of course, greatly exaggerated when compared with the temperature distribution obtained under actual operating conditions, an example of which is shown in figure 15. It is evident, however, that the temperatures along the central 12 inches of the tube are not appreciably affected by the end losses in either case. Therefore, by taking the central 12 inches of the tube as the test section, the effect of end losses is reduced to a minimum.

Measurements of electrical resistance of the tube. - The power input to any section of the tube may, of course, be calculated by the I^2R law if the electrical resistance of the tube is known. The resistance of the tube per inch length was therefore obtained over a range of operating temperatures from the ammeter and the voltmeter readings and the length of the tube after the power factor of the tube was checked with an oscillograph and found to be unity. The results of the foregoing computations are shown in figure 5 where the resistance of the tube per inch length is shown as a function of temperature. The results of resistance measurements made with a Kelvin bridge using a sample tube of the same material agreed within approximately 1.5 percent with the alternating-current results.

APPENDIX B

CALCULATION OF INSIDE-TUBE-WALL TEMPERATURES

In order to evaluate the heat-transfer coefficient between the inside-tube-wall surface and the liquid, a relation for calculating the inside-tube-wall temperature from the measured outside-tube-wall temperature was obtained.

It is known that if all the heat passes the full thickness x of the tube wall, then the temperature drop through the wall could be calculated from the following familiar equation:

$$q = \frac{k_s A_m}{x} (t_o - t_i) \tag{1}$$

In view of the fact that the heat is produced by an electric current flowing through the tube and, hence all of the heat going to the liquid does not pass the full thickness of the tube, equation (1) is not valid for this application.

A relation similar to equation (1), however, giving the actual temperature drop through the tube wall can be obtained as follows:

Assuming that: (a) the heat is produced in the tube uniformly across the tube-wall thickness; and (b) the heat flow is in only one direction (toward the liquid), for a section of unit length the heat generated from the outer radius y_o to any other radius y is as follows:

$$q = \beta \frac{\pi E^2}{\rho} (y_o{}^2 - y^2) \tag{2}$$

and the heat conducted is:

$$q = 2\pi\, y\, k_s \frac{dt}{dy} \tag{3}$$

Combination of equations (2) and (3) results in the following:

$$\beta \frac{\pi E^2}{\rho} (y_o{}^2 - y^2) = 2\pi\, y\, k_s \frac{dt}{dy}$$

or separating the variables and rewriting:

$$dt = \beta \frac{\pi E^2}{2\pi\, \rho\, k_s} \left(\frac{y_o{}^2 - y^2}{y}\right) dy$$

Integrating between the limits of y_i and y_o and t_i and t_o:

$$t_o - t_i = \beta \frac{\pi E^2 y_o^2 \log_e \left(\frac{y_o}{y_i}\right)}{2\pi \rho k_s} - \beta \frac{\pi E^2 (y_o^2 - y_i^2)}{4\pi \rho k_s} \qquad (4)$$

But the total heat produced can be expressed as

$$q = \beta \frac{\pi E^2}{\rho} (y_o^2 - y_i^2)$$

and substituting from this expression into equation (4) results in the following:

$$t_o - t_i = \frac{q y_o^2 \log_e \left(\frac{y_o}{y_i}\right)}{2\pi k_s (y_o^2 - y_i^2)} - \frac{q}{4\pi k_s}$$

or rewriting

$$q = \frac{2\pi k_s (t_o - t_i)}{\log_e \left(\frac{y_o}{y_i}\right) \left[\frac{y_o^2}{y_o^2 - y_i^2} - \frac{1}{2 \log_e \left(\frac{y_o}{y_i}\right)}\right]} \qquad (5)$$

Substituting the values of y_o (0.0208 ft) and y_i (0.0182 ft) in the bracketed term and noting that $\frac{2\pi}{\log_e \left(\frac{y_o}{y_i}\right)}$ is equal to $\frac{A_m}{x}$ results in the following expression, which gives the theoretical temperature drop through the tube wall:

$$q = \frac{k_s A_m}{x} \left(\frac{t_o - t_i}{0.525}\right) \qquad (6)$$

The constant 0.525 in equation (6) was rounded off to 0.5 for the actual calculations because it was felt that the simplifying assumptions used in the derivations did not justify additional significant places.

REFERENCES

1. McAdams, William H.: Heat Transmission. McGraw-Hill Book Co., Inc., 2d ed., 1942, pp. 164, 167-168, 178-181.

2. Cragoe, C. S.: Properties of Ethylene Glycol and Its Aqueous Solutions. Cooperative Fuel Res. Committee, CRC, July 1943.

3. Manganiello, E. J., and Stalder, J. R.: Heat-Transfer Tests of Several Engine Coolants. NACA ARR No. E5B06, 1945.

4. Thum, Ernest E.: The Book of Stainless Steels. Am. Soc. Metals, 2d ed., 1935, p. 231.

5. Anon.: The Fabrication of Republic Enduro Stainless Steels. Republic Steel Corp. (Cleveland), 1942, p. 37.

6. Barnes, Howard T.: The Heat Capacity of Chemical Compounds in the Liquid State. Vol. V of International Critical Tables of Numerical Data, Physics, Chemistry and Technology, Nat. Res. Council, Edward Washburn ed., McGraw-Hill Book Co., Inc., 1929, pp. 106-113.

7. Barrat, T., and Nettleton, H. R.: Thermal Conductivity of Liquids and Solids. Non-Metallic Liquids. Vol. V of International Critical Tables of Numerical Data, Physics, Chemistry and Technology, Nat. Res. Council, Edward Washburn ed., McGraw-Hill Book Co., Inc., 1929, pp. 226-229.

8. Giordani, F.: Viscosity of Pure Liquids. Vol. VII of International Critical Tables of Numerical Data, Physics, Chemistry and Technology, Nat. Res. Council, Edward Washburn ed., McGraw-Hill Book Co., Inc., 1930, pp. 211-224.

TABLE I - SUMMARY OF DATA AND RESULTS

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Run	Tube current I (amp)	Heat rate, Btu/sec: Input: Test section q (center 12 in.)	Heat rate, Btu/sec: Input: Full section q' (22.75 in.)	Heat rate, Btu/sec: Rejected to liquid q_r	Liquid-flow rate W (lb/sec)	Liquid temperature (°F): Average t	Liquid temperature (°F): Rise Δt	Liquid pressure p (lb/sq in. absolute)	Average tube-wall temperature of center 12 inches of test section (°F): Outside t_o	Average tube-wall temperature of center 12 inches of test section (°F): Inside t_i	Heat-transfer coefficient h (Btu)/(sec)(sq ft)(°F)	Prandtl number $c\mu/k$	Reynolds number DG/μ	Nusselt number hD/k	Stanton number h/cG
Test with variable average liquid temperature; liquid, water															
126	660	3.00	5.65	5.19	0.33	122.3	15.9	56	190	178	0.47	3.5	31,000	167.2	0.00151
127	660	3.02	5.67	5.15	.32	136.5	16.0	57	203	191	.49	3.1	34,700	170.3	.00158
128	653	2.98	5.60	5.05	.32	153.8	15.9	56	218	206	.50	2.6	39,400	172.7	.00165
129	648	2.95	5.56	5.09	.31	160.0	16.2	56	228	216	.46	2.5	40,800	157.7	.00153
130	643	2.92	5.50	4.93	.31	173.0	16.0	59	235	223	.51	2.3	44,100	174.0	.00172
131	643	2.94	5.55	5.26	.33	187.5	15.8	58	248	236	.53	2.1	52,400	178.2	.00166
132	686	3.31	6.25	5.60	.32	152.0	17.7	59	223	210	.50	2.7	38,700	173.3	.00165
133	650	2.95	5.56	4.90	.32	148.8	15.5	59	213	201	.49	2.8	37,700	169.7	.00162
142	677	3.21	6.11	5.39	.18	114.5	29.5	62	222	209	.30	3.9	16,200	106.2	.00171
143	674	3.21	6.07	5.41	.19	129.3	29.3	57	233	220	.31	3.3	18,700	108.4	.00175
144	667	3.16	5.97	5.35	.18	143.8	29.4	54	244	231	.32	2.9	20,800	109.1	.00184
145	670	3.21	6.06	5.53	.19	157.5	29.6	58	261	248	.31	2.6	23,900	106.1	.00173
146	665	3.19	6.02	5.37	.18	173.8	29.8	63	275	262	.31	2.3	26,000	106.6	.00179
147	660	3.17	5.97	5.29	.18	189.5	29.6	64	290	277	.31	2.0	28,600	105.3	.00181
148	674	3.19	6.01	6.00	.20	115.0	30.5	62	220	207	.30	3.8	17,400	107.9	.00160
149	674	3.21	6.07	5.58	.19	131.3	29.1	65	232	219	.32	3.2	19,700	111.9	.00175
150	672	3.20	6.04	5.46	.19	146.3	29.2	63	244	231	.33	2.8	21,800	114.1	.00185
151	667	3.18	6.01	5.43	.19	159.0	29.0	60	255	242	.33	2.5	24,100	114.4	.00184
152	665	3.18	5.99	5.28	.18	172.8	29.3	62	270	257	.33	2.3	25,800	112.0	.00191
153	667	3.24	6.08	5.37	.18	190.0	29.4	63	290	277	.32	2.0	29,200	109.2	.00183
154	665	3.17	5.96	5.35	.19	174.0	28.4	60	264	251	.36	2.3	27,200	120.7	.00199
155	670	3.18	6.00	5.35	.19	145.0	28.6	62	245	232	.32	2.9	21,600	110.2	.00179
156	677	3.22	6.06	5.68	.20	113.3	28.7	63	225	212	.29	3.9	17,300	101.7	.00153
157	684	3.21	6.05	5.73	.58	125.5	10.0	63	180	167	.69	3.4	56,200	242.0	.00125
158	682	3.18	6.01	5.72	.58	125.5	10.0	15	179	166	.69	3.4	56,200	243.8	.00125
160	806	4.46	8.44	8.06	.58	103.8	14.0	63	182	163	.65	4.4	45,600	235.0	.00118
161	802	4.44	8.38	7.90	.57	118.5	13.8	63	194	176	.68	3.7	52,400	241.8	.00124
162	797	4.41	8.33	7.85	.57	129.3	13.7	63	203	185	.70	3.3	58,000	243.7	.00128
163	799	4.47	8.43	7.94	.57	144.3	13.9	63	217	199	.72	2.9	65,900	248.5	.00131
164	797	4.47	8.41	7.91	.57	158.5	13.8	63	229	211	.75	2.5	73,600	257.1	.00136
165	785	4.36	8.23	7.59	.56	173.3	13.6	63	240	222	.77	2.3	80,200	262.4	.00144
166	773	4.25	8.06	7.43	.56	182.8	13.3	63	247	230	.78	2.1	85,500	264.0	.00145
167	775	4.31	8.11	7.53	.56	198.0	13.4	66	262	245	.80	1.9	94,600	269.2	.00149
168	775	4.28	8.09	7.68	.89	196.5	8.6	67	249	232	1.05	1.9	149,400	352.9	.00123
169	778	4.28	8.12	7.66	.88	183.5	8.7	65	238	221	1.02	2.1	135,900	343.3	.00120
170	785	4.33	8.25	7.82	.89	173.5	8.8	65	228	210	1.03	2.3	127,900	349.4	.00121
171	794	4.41	8.36	7.88	.89	158.0	8.9	64	215	197	.99	2.6	113,400	339.3	.00117
172	797	4.40	8.34	8.03	.91	143.0	8.8	67	201	183	.97	2.9	103,000	335.5	.00112
173	806	4.47	8.41	8.06	.90	125.5	8.9	63	185	166	.95	3.4	88,100	333.5	.00110
174	811	4.47	8.46	8.14	.90	100.5	9.0	70	165	146	.86	4.5	68,900	310.8	.00100
192	778	4.33	8.20	7.85	.20	118.5	40.1	68	260	243	.31	3.7	17,900	108.2	.00166
193	770	4.25	8.07	7.61	.20	126.5	38.9	67	263	246	.31	3.4	19,300	109.4	.00166
194	773	4.31	8.15	7.73	.20	143.0	39.5	69	274	257	.33	2.9	22,200	113.9	.00176
195	768	4.28	8.10	7.73	.20	157.0	39.4	68	285	268	.34	2.6	24,900	114.8	.00181
196	761	4.22	8.03	6.93	.18	170.5	39.1	68	295	278	.34	2.3	24,900	116.1	.00201
197	761	4.23	8.06	6.92	.18	176.5	39.0	68	301	284	.34	2.2	25,900	115.8	.00200

TABLE I - SUMMARY OF DATA AND RESULTS - Continued

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Run	Tube current I (amp)	Heat rate, Btu/sec			Liquid-flow rate W (lb/sec)	Liquid temperature (°F)		Liquid pressure p (lb/sq in. absolute)	Average tube-wall temperature of center 12 inches of test section (°F)		Heat-transfer coefficient h (Btu)/(sec)(sq ft)(°F)	Prandtl number $c\mu/k$	Reynolds number DG/μ	Nusselt number hD/k	Stanton number h/cG
		Input		Rejected to liquid q_r		Average t	Rise Δt		Outside t_o	Inside t_i					
		Test section q (center 12 in.)	Full section q' (22.75 in.)												
Test with variable average liquid temperature; liquid, water - Concluded															
309	782	4.35	8.27	7.53	0.20	94.3	38.7	69	246	228	0.28	4.9	13,900	104.4	0.00150
310	782	4.38	8.28	7.57	.20	119.3	38.9	69	258	240	.32	3.7	18,000	112.0	.00172
311	782	4.39	8.31	7.66	.20	130.3	39.3	69	266	248	.33	3.3	19,900	114.0	.00177
312	773	4.30	8.15	7.56	.20	144.3	38.5	69	273	256	.34	2.9	22,600	116.2	.00181
313	766	4.25	8.06	7.55	.20	156.5	38.7	69	284	267	.34	2.6	24,800	114.7	.00182
314	763	4.25	8.07	6.98	.18	176.3	39.8	69	300	283	.35	2.2	25,700	117.9	.00208
333	742	4.01	7.45	8.24	.83	247.9	9.9	70	295	279	1.14	1.4	187,400	377.4	.00142
334	749	4.04	7.50	7.92	.84	223.5	9.4	70	273	257	1.05	1.6	165,100	349.2	.00130
335	768	4.20	7.77	7.98	.83	197.8	9.6	70	251	234	1.01	1.9	139,700	338.6	.00127
336	773	4.20	7.80	8.09	.83	173.3	9.7	70	229	212	.96	2.3	119,600	323.9	.00120
337	773	4.15	7.72	8.06	.84	149.8	9.6	70	206	189	.93	2.7	100,700	318.9	.00116
338	780	4.18	7.76	8.11	.83	123.5	9.8	70	183	166	.86	3.5	79,300	304.2	.00109
339	794	4.28	7.89	8.05	.83	94.9	9.8	70	160	142	.79	4.9	59,200	290.3	.00100
366	751	4.03	7.51	7.37	.67	197.7	11.0	68	256	240	.84	1.9	112,500	280.8	.00131
367	754	4.05	7.56	7.79	.67	197.3	11.6	15	256	240	.84	1.9	112,500	281.3	.00130
368	763	4.11	7.72	7.81	.68	172.3	11.5	68	235	218	.78	2.3	96,200	266.0	.00120
369	761	4.09	7.68	7.84	.67	171.9	11.7	15	234	218	.78	2.3	94,900	264.9	.00122
370	768	4.12	7.74	7.91	.67	146.4	11.8	68	213	196	.73	2.8	78,300	250.9	.00114
371	780	4.20	7.87	7.81	.68	121.7	11.5	68	190	173	.72	3.6	64,000	252.8	.00111
372	785	4.20	7.91	8.00	.67	98.9	11.9	68	171	153	.68	4.6	50,300	247.2	.00106
Test with variable liquid-flow rate; liquid, water															
119	648	2.93	5.52	4.89	0.32	152.0	15.4	57	214	202	0.51	2.7	38,700	175.8	0.00168
120	650	2.93	5.53	4.86	.48	149.5	10.1	59	200	188	.68	2.7	57,600	232.2	.00148
121	653	2.94	5.53	4.78	.65	150.5	7.4	57	194	182	.83	2.7	78,500	284.0	.00133
122	653	2.94	5.51	4.67	.81	151.0	5.7	57	189	177	.98	2.7	98,600	336.3	.00126
123	660	2.99	5.61	4.60	.98	149.0	4.7	60	185	173	1.10	2.8	116,600	379.7	.00117
124	660	2.99	5.62	4.57	1.14	149.8	4.0	57	184	172	1.20	2.7	137,500	414.1	.00110
125	655	2.94	5.53	4.34	1.28	148.5	3.4	56	181	169	1.31	2.8	152,400	449.1	.00107
134	667	3.16	5.98	5.47	.19	149.5	29.1	53	243	230	.34	2.7	22,600	117.1	.00189
135	674	3.18	6.03	5.57	.32	148.5	17.3	59	219	206	.49	2.8	38,200	167.8	.00159
136	679	3.21	6.08	5.68	.49	149.8	11.7	55	205	192	.66	2.7	58,600	228.1	.00142
137	684	3.24	6.11	5.76	.65	149.5	8.8	55	198	185	.80	2.7	78,200	274.9	.00128
138	679	3.18	6.02	5.71	.82	149.3	7.0	53	192	179	.93	2.8	97,500	319.9	.00119
139	684	3.23	6.09	5.78	.98	149.5	5.9	53	190	177	1.05	2.7	117,000	360.1	.00112
140	682	3.21	6.07	5.77	.82	148.5	7.1	53	193	180	.90	2.8	97,000	309.0	.00115
141	665	3.14	5.92	5.43	.18	149.0	30.0	53	244	231	.33	2.8	21,800	114.3	.00188
175	761	4.22	8.04	6.91	.18	179.5	38.3	62	296	279	.37	2.2	27,000	124.6	.00214
176	768	4.24	8.03	7.20	.31	179.5	22.9	62	267	250	.52	2.2	47,000	176.7	.00173
177	775	4.28	8.12	7.54	.48	178.5	15.6	62	251	234	.68	2.2	71,600	231.1	.00147
178	778	4.29	8.12	7.70	.64	178.5	12.0	61	241	224	.82	2.2	95,300	278.1	.00133
179	785	4.34	8.23	7.81	.89	178.0	8.7	61	233	215	1.02	2.2	132,200	345.7	.00119
180	785	4.34	8.22	7.83	.98	177.0	8.0	61	231	213	1.05	2.2	143,700	356.0	.00112
181	787	4.36	8.27	7.89	1.14	177.5	6.9	61	228	210	1.18	2.2	168,400	398.5	.00108

TABLE I - SUMMARY OF DATA AND RESULTS - Continued

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Run	Tube current I (amp)	Heat rate, Btu/sec: Input: Test section q (center 12 in.)	Heat rate, Btu/sec: Input: Full section q' (22.75 in.)	Heat rate, Btu/sec: Rejected to liquid q_r	Liquid-flow rate W (lb/sec)	Liquid temperature (°F): Average t	Liquid temperature (°F): Rise Δt	Liquid pressure p (lb/sq in. absolute)	Average tube-wall temperature of center 12 inches of test section (°F): Outside t_o	Average tube-wall temperature of center 12 inches of test section (°F): Inside t_i	Heat-transfer coefficient h (Btu)/(sec)(sq ft)(°F)	Prandtl number $c\mu/k$	Reynolds number DG/μ	Nusselt number hD/k	Stanton number h/cG
Test with variable liquid-flow rate; liquid, water - Concluded															
182	790	4.38	8.29	7.92	1.29	178.5	6.1	61	226	208	1.28	2.2	191,400	431.4	0.00103
183	775	4.30	8.13	7.64	.20	127.1	37.7	62	260	243	.32	3.4	20,100	113.9	.00165
184	785	4.33	8.18	7.72	.33	128.5	23.6	62	228	210	.46	3.3	32,900	163.0	.00146
185	794	4.39	8.28	7.91	.50	128.5	15.8	64	208	190	.63	3.3	50,200	219.7	.00132
186	797	4.41	8.31	7.98	.58	127.5	13.9	64	202	184	.68	3.4	57,400	238.9	.00123
187	797	4.40	8.31	8.05	.66	128.3	12.2	64	198	180	.74	3.3	66,000	260.4	.00117
188	802	4.43	8.37	8.13	.83	127.3	9.8	64	190	172	.87	3.4	82,200	305.7	.00110
189	809	4.50	8.48	8.33	.99	127.3	8.5	64	186	167	.98	3.4	97,700	342.2	.00104
190	814	4.54	9.18	8.36	1.15	127.0	7.3	63	183	164	1.08	3.4	113,800	380.7	.00098
191	814	4.54	8.55	8.43	1.31	129.3	6.5	63	181	162	1.20	3.3	132,100	420.8	.00096
340	749	4.02	7.49	7.78	.21	148.3	36.8	69	265	249	.35	2.8	25,100	120.6	.00173
341	749	4.02	7.48	7.77	.21	147.5	36.4	69	263	247	.35	2.8	25,200	121.6	.00171
342	758	4.07	7.58	8.20	.33	147.3	24.9	69	237	220	.49	2.8	38,800	167.1	.00155
343	761	4.06	7.60	8.66	.50	150.0	17.3	69	222	205	.64	2.7	60,200	221.1	.00134
344	763	4.06	7.60	8.80	.66	149.5	13.3	69	212	195	.77	2.8	78,900	264.7	.00122
345	773	4.15	7.68	8.63	.82	148.5	10.5	69	206	189	.91	2.8	97,500	312.6	.00115
346	782	4.25	7.85	8.53	1.00	148.0	8.5	69	201	183	1.05	2.8	118,400	362.7	.00110
347	778	4.18	7.75	8.81	1.15	148.3	7.7	69	197	180	1.15	2.8	135.500	395.5	.00105
348	780	4.21	7.77	8.82	1.29	149.8	6.8	69	197	180	1.25	2.7	154,800	431.4	.00101
350	734	3.90	7.29	7.49	.17	150.3	43.2	68	280	264	.30	2.7	20,900	102.9	.00181
351	737	3.93	7.36	7.68	.17	148.5	44.4	49	281	265	.29	2.8	20,600	101.0	.00175
352	751	3.99	7.47	7.55	.34	150.3	22.2	68	236	220	.50	2.7	40,900	171.3	.00154
353	751	3.99	7.46	7.63	.34	150.3	22.5	45	236	220	.50	2.7	40,900	171.3	.00154
354	758	4.03	7.55	7.59	.51	147.3	15.0	68	218	201	.65	2.8	59,400	223.8	.00134
355	758	4.03	7.55	7.61	.50	148.0	15.1	33	219	202	.65	2.8	59,400	222.6	.00135
356	754	3.95	7.38	7.30	.68	148.5	10.8	68	207	191	.82	2.8	79,900	280.7	.00127
357	754	3.95	7.39	7.33	.67	149.3	10.9	15	209	193	.81	2.8	80,200	276.8	.00126
358	758	3.99	7.45	7.38	.83	147.5	8.9	68	202	185	.91	2.8	97,800	314.3	.00114
359	758	3.99	7.48	7.36	.83	149.0	8.9	15	204	187	.92	2.8	96,900	315.7	.00116
360	763	4.03	7.52	7.15	1.00	146.8	7.2	68	199	182	1.00	2.8	116,200	344.8	.00105
361	763	4.03	7.54	7.23	1.00	146.8	7.3	15	199	182	1.00	2.8	116,200	343.9	.00105
362	763	4.03	7.55	7.24	1.16	149.3	6.3	68	196	180	1.16	2.8	138,500	398.0	.00105
363	761	4.00	7.49	7.29	1.15	148.3	6.3	15	196	180	1.12	2.8	136,100	384.5	.00102
364	761	4.00	7.50	7.16	1.31	148.8	5.5	68	194	178	1.21	2.8	155,000	417.2	.00097
365	763	4.02	7.52	7.17	1.31	148.5	5.5	15	194	177	1.22	2.8	155,200	419.8	.00098
495	751	3.88	7.32	6.20	2.48	150.5	2.5	68	184	168	1.92	2.7	299,600	661.1	.00081
496	754	3.90	7.36	6.60	2.20	150.5	3.0	68	186	170	1.81	2.7	265,800	623.3	.00086
497	749	3.86	7.29	6.53	1.92	151.0	3.4	68	188	172	1.64	2.7	232,900	564.9	.00089
498	749	3.86	7.28	6.42	1.69	149.5	3.8	68	189	173	1.47	2.7	201,800	504.3	.00091
499	746	3.85	7.22	6.26	1.36	152.5	4.6	68	192	176	1.41	2.7	166,600	484.2	.00108
500	751	3.90	7.31	6.51	1.21	154.0	5.4	68	191	175	1.31	2.6	149,700	449.6	.00113
501	751	3.91	7.33	6.80	1.00	151.0	6.8	68	198	182	1.10	2.7	121,300	377.0	.00115
502	756	3.98	7.42	6.96	.75	150.5	9.3	68	207	191	.87	2.7	90,400	299.1	.00121

TABLE I - SUMMARY OF DATA AND RESULTS - Continued

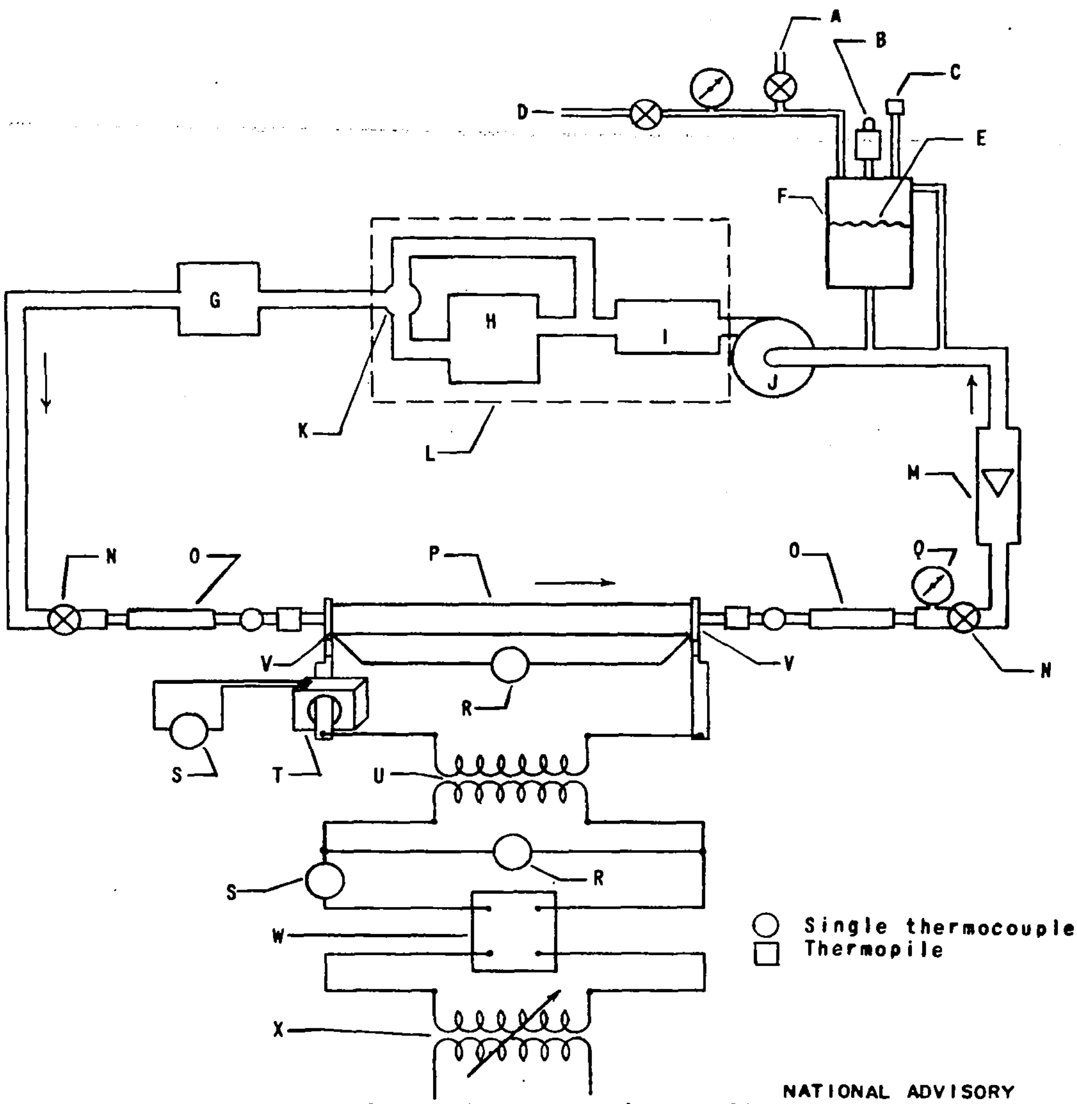
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Run	Tube current I (amp)	Heat rate, Btu/sec: Input: Test section q (center 12 in.)	Heat rate, Btu/sec: Input: Full section q' (22.75 in.)	Heat rate, Btu/sec: Rejected to liquid q_r	Liquid-flow rate W (lb/sec)	Liquid temperature (°F): Average t	Liquid temperature (°F): Rise Δt	Liquid pressure p (lb/sq in. absolute)	Average tube-wall temperature of center 12 inches of test section (°F): Outside t_o	Average tube-wall temperature of center 12 inches of test section (°F): Inside t_i	Heat-transfer coefficient h (Btu)/(sec)(sq ft)(°F)	Prandtl number cμ/k	Reynolds number DG/μ	Nusselt number hD/k	Stanton number h/cG
Test with variable heat input; liquid, water															
323	331	0.75	1.41	1.13	0.20	150.8	5.7	69	175	172	0.31	2.7	23,800	105.6	0.00165
324	492	1.68	3.17	2.72	.20	150.8	13.8	69	205	198	.31	2.7	24,000	106.3	.00163
325	614	2.67	5.03	4.51	.20	149.0	22.9	69	236	226	.30	2.8	23,400	104.7	.00159
326	739	3.94	7.47	6.81	.20	150.0	34.4	69	274	258	.32	2.7	23,900	109.6	.00168
327	773	4.33	8.15	7.51	.20	149.3	37.9	69	284	267	.32	2.8	23,800	110.5	.00168
Test with variable average liquid temperature; liquid, AN-E-2 ethylene glycol															
481	394	1.08	2.04	1.81	0.67	123.1	4.4	68	209	205	0.12	59.9	5,000	99.2	0.00031
482	396	1.10	2.06	1.80	.67	150.0	4.3	68	214	210	.16	41.8	7,700	143.6	.00040
483	394	1.09	2.05	1.71	.67	171.5	4.0	68	227	223	.19	32.3	10,500	172.9	.00046
484	389	1.08	2.03	1.74	.67	198.3	4.0	68	249	245	.20	25.5	14,500	197.5	.00048
515	298	.62	1.13	.84	.97	199.5	1.3	68	220	217	.31	25.2	23,100	291.0	.00051
516	298	.62	1.14	.98	.96	191.5	1.6	68	213	210	.28	26.9	21,000	257.6	.00047
517	298	.62	1.14	1.02	.95	180.0	1.7	68	204	201	.25	29.8	18,000	228.8	.00043
518	298	.61	1.14	1.02	.96	171.5	1.7	68	196	193	.25	32.4	16,200	221.8	.00043
519	298	.61	1.14	1.03	.97	163.0	1.7	68	188	185	.24	35.6	14,500	209.5	.00041
520	298	.61	1.14	1.01	.96	153.0	1.7	68	180	177	.21	40.1	12,400	185.4	.00037
521	302	.62	1.15	1.05	.95	140.5	1.8	68	170	167	.20	47.0	10,300	172.6	.00035
522	302	.62	1.15	1.08	.94	132.0	1.9	68	165	162	.18	53.0	8,800	153.2	.00032
Test with variable liquid-flow rate; liquid, AN-E-2 ethylene glycol															
476	389	1.07	2.03	1.74	0.67	199.5	4.0	68	246	242	0.22	25.2	14,900	215.0	0.00052
477	384	1.07	2.03	1.72	.33	200.5	7.8	68	280	276	.12	25.1	7,500	120.4	.00057
478	391	1.08	2.04	1.73	1.00	197.9	2.6	68	235	231	.29	25.7	21,900	282.5	.00046
479	394	1.09	2.05	1.69	1.25	197.7	2.1	68	230	226	.35	25.8	26,900	335.9	.00045
480	389	1.07	2.03	1.73	.67	198.7	4.0	68	247	243	.21	25.5	14,600	206.3	.00050
503	276	.52	.94	1.32	2.51	150.5	.8	68	162	160	.49	41.3	31,500	426.2	.00033
504	278	.53	.95	1.18	2.25	149.5	.8	68	162	160	.44	41.9	27,800	379.0	.00033
505	278	.53	.95	1.16	1.99	150.5	.9	68	164	162	.43	41.3	25,000	369.5	.00036
506	278	.53	.95	1.08	1.70	150.5	1.0	68	166	164	.35	41.3	21,400	307.5	.00034
507	278	.53	.95	1.05	1.42	149.5	1.2	68	167	165	.30	41.9	17,600	262.4	.00035
508	278	.53	.95	.91	1.14	151.0	1.3	68	171	169	.26	41.1	14,400	227.7	.00038
509	276	.52	.94	.84	1.45	151.5	.9	68	168	166	.33	40.9	18,400	285.6	.00038
510	276	.52	.94	.88	1.28	150.0	1.1	68	168	166	.29	41.5	15,900	247.4	.00038
511	276	.52	.94	.87	1.09	151.0	1.3	68	171	169	.25	41.1	13,800	216.6	.00038
512	276	.52	.94	.85	.89	149.0	1.5	68	174	172	.20	42.1	11,000	174.2	.00037
513	276	.52	.94	.87	.71	152.0	1.9	68	181	179	.17	40.7	9,100	150.9	.00040
514	276	.53	.94	.78	.49	151.5	2.6	68	193	191	.12	40.9	6,200	101.2	.00041
Test with variable average liquid temperature; liquid, nominal (by volume) 70 percent-30 percent glycol-water															
420	322	0.74	1.38	1.04	0.67	224.8	2.0	68	247	244	0.33	9.8	36,800	238.2	0.00065
421	322	.73	1.37	1.20	.67	199.3	2.3	68	224	221	.30	11.8	29,500	209.7	.00061
422	326	.74	1.39	1.23	.67	174.8	2.4	68	201	198	.28	14.6	23,000	190.3	.00057
423	326	.73	1.38	1.22	.67	149.0	2.5	68	179	176	.24	18.7	17,200	160.8	.00051
424	329	.73	1.39	1.29	.67	124.8	2.7	68	159	156	.21	24.8	12,500	137.4	.00045
425	331	.74	1.38	1.24	.67	100.0	2.6	68	140	137	.17	34.5	8,600	112.3	.00037

TABLE I - SUMMARY OF DATA AND RESULTS - Concluded

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Run	Tube current I (amp)	Heat rate, Btu/sec: Input: Test section q (center 12 in.)	Heat rate, Btu/sec: Input: Full section q' (22.75 in.)	Heat rate, Btu/sec: Rejected to liquid q_r	Liquid-flow rate W (lb/sec)	Liquid temperature (°F): Average t	Liquid temperature (°F): Rise Δt	Liquid pressure p (lb/sq in. absolute)	Average tube-wall temperature of center 12 inches of test section (°F): Outside t_o	Average tube-wall temperature of center 12 inches of test section (°F): Inside t_i	Heat-transfer coefficient h (Btu)/(sec)(sq ft)(°F)	Prandtl number $c\mu/k$	Reynolds number DG/μ	Nusselt number hD/k	Stanton number h/cG
Test with variable liquid-flow rate; liquid, nominal (by volume) 70 percent-30 percent glycol-water															
412	324	0.73	1.38	1.18	0.25	150.9	6.2	68	209	206	0.12	18.3	6,700	78.2	0.00066
413	326	.74	1.38	1.20	.34	150.0	4.8	68	195	192	.15	18.5	8,800	103.6	.00063
414	326	.73	1.38	1.26	.50	149.4	3.4	68	183	180	.21	18.6	12,900	143.0	.00059
415	326	.73	1.38	1.23	.68	148.9	2.5	68	176	173	.26	18.7	17,300	177.1	.00054
416	326	.73	1.38	1.26	.83	149.5	2.0	68	173	170	.31	18.6	21,600	211.7	.00052
417	329	.74	1.38	1.27	1.01	149.8	1.7	68	170	167	.39	18.5	26,200	261.3	.00054
418	329	.74	1.38	1.34	1.18	149.3	1.5	68	167	164	.43	18.6	30,500	288.9	.00051
419	331	.75	1.39	1.25	1.33	149.5	1.3	68	167	164	.45	18.6	34,300	302.0	.00048
Test with variable average liquid temperature; liquid, nominal (by volume) 30 percent-70 percent glycol-water															
457	626	2.72	5.15	4.73	0.67	150.5	7.8	68	203	192	0.58	6.2	43,200	264.7	0.00100
458	619	2.76	5.07	4.75	.66	220.8	7.7	68	268	257	.66	3.6	74,900	305.0	.00112
459	622	2.75	5.07	4.73	.67	197.5	7.6	68	249	238	.60	4.2	64,900	273.3	.00101
460	622	2.72	5.07	4.73	.67	173.5	7.7	68	226	215	.58	5.0	52,100	262.4	.00098
461	626	2.72	5.06	4.61	.67	144.0	7.6	68	202	191	.51	6.6	40,500	234.1	.00088
462	629	2.72	5.07	4.59	.66	121.8	7.7	68	183	172	.47	8.4	31,600	219.3	.00082
463	631	2.71	5.07	4.62	.67	100.0	7.8	68	165	154	.44	11.0	24,300	205.2	.00077
464	626	2.73	5.06	4.82	.67	149.5	7.9	68	204	193	.55	6.2	42,600	255.0	.00095
465	619	2.76	5.07	4.74	.67	222.8	7.6	68	271	260	.65	3.6	77,000	299.1	.00109
466	619	2.73	5.07	4.81	.67	197.3	7.8	68	248	237	.60	4.2	64,600	271.7	.00101
467	622	2.72	5.06	4.71	.67	171.8	7.7	68	226	215	.55	5.1	52,400	249.3	.00094
468	629	2.72	5.06	4.72	.67	123.5	7.9	68	184	173	.48	8.3	32,400	223.7	.00084
469	636	2.75	5.11	4.71	.67	96.8	7.9	68	165	153	.43	11.5	23,200	199.6	.00076
470	626	2.73	5.06	4.79	.66	146.8	8.0	68	204	193	.52	6.4	41,400	239.7	.00090
Test with variable liquid-flow rate; liquid, nominal (by volume) 30 percent-70 percent glycol-water															
440	614	2.70	5.05	4.79	0.25	149.0	20.8	68	260	249	0.24	6.3	16,100	108.1	0.00109
441	617	2.70	5.04	4.78	.34	148.0	15.6	68	244	233	.28	6.3	21,400	127.3	.00095
442	622	2.71	5.03	4.69	.50	148.5	10.3	68	223	212	.37	6.3	31,700	172.3	.00085
443	622	2.70	5.04	4.86	.67	149.0	8.0	68	213	202	.45	6.3	42,500	205.5	.00077
444	626	2.72	5.06	4.79	.84	147.2	6.3	68	203	192	.54	6.4	52,700	247.1	.00074
445	629	2.73	5.07	4.76	1.00	147.3	5.2	68	194	183	.68	6.4	62,600	310.9	.00078
446	629	2.73	5.05	4.85	1.17	146.5	4.6	68	190	179	.74	6.4	72,800	340.3	.00073
447	629	2.72	5.07	4.91	1.33	149.5	4.1	68	190	179	.83	6.2	85,400	381.2	.00071
471	626	2.73	5.06	4.82	.67	147.0	8.0	68	204	193	.52	6.4	41,600	239.1	.00090
472	629	2.72	5.07	5.13	1.33	147.0	4.2	68	167	176	.83	6.4	83,300	382.4	.00072
473	619	2.73	5.07	4.80	.25	144.5	21.2	68	250	239	.25	6.6	15,200	116.8	.00115
Test with variable average liquid temperature; liquid, commercial butanol															
529	384	1.04	1.97	1.99	0.81	202.5	3.2	68	236	232	0.31	11.7	70,300	430.1	0.00052
530	390	1.07	2.00	1.98	.81	182.5	3.3	68	220	216	.28	13.8	56,300	386.9	.00049
531	394	1.08	2.01	1.99	.81	160.5	3.5	68	202	198	.25	16.6	44,700	350.4	.00046
532	396	1.08	2.03	2.07	.81	144.0	3.8	68	188	183	.24	19.1	37,600	328.4	.00046
533	398	1.08	2.04	2.06	.80	121.0	4.0	68	170	165	.21	23.4	28,800	293.7	.00043
Test with variable liquid-flow rate; liquid, commercial butanol															
523	413	1.21	2.27	2.29	0.35	151.0	9.5	68	239	234	0.13	18.0	17,600	175.3	0.00056
524	422	1.25	2.32	2.32	.52	152.3	6.5	68	218	213	.18	17.8	26,300	250.6	.00053
525	422	1.24	2.33	2.36	.74	154.0	4.6	68	204	199	.24	17.5	38,100	334.0	.00049
526	422	1.24	2.32	2.35	.86	153.5	4.0	68	199	194	.27	17.6	44,200	373.8	.00047
527	425	1.25	2.34	2.34	.99	153.0	3.5	68	194	189	.30	17.7	50,200	419.1	.00046
528	425	1.24	2.34	2.41	1.16	152.0	3.0	68	190	185	.33	17.8	58,500	459.0	.00043



A Air bleed line
B Pressure-relief valve
C Filling cap
D Compressed-air connection
E Liquid level
F Expansion tank
G Filter
H Cooler
I Electric heater
J Pump
K Temperature regulator
L Heating and cooling blending unit
M Rotameter
N Throttle valve
O Electrical-insulating coupling
P Heater tube
Q Pressure gage
R Voltmeter
S Ammeter
T 240:1 current transformer
U 20:1 power transformer
V Clamp-type copper connectors
W Voltage regulator
X Autotransformer

Figure 1. - Schematic diagram of heater-tube setup and associated equipment.

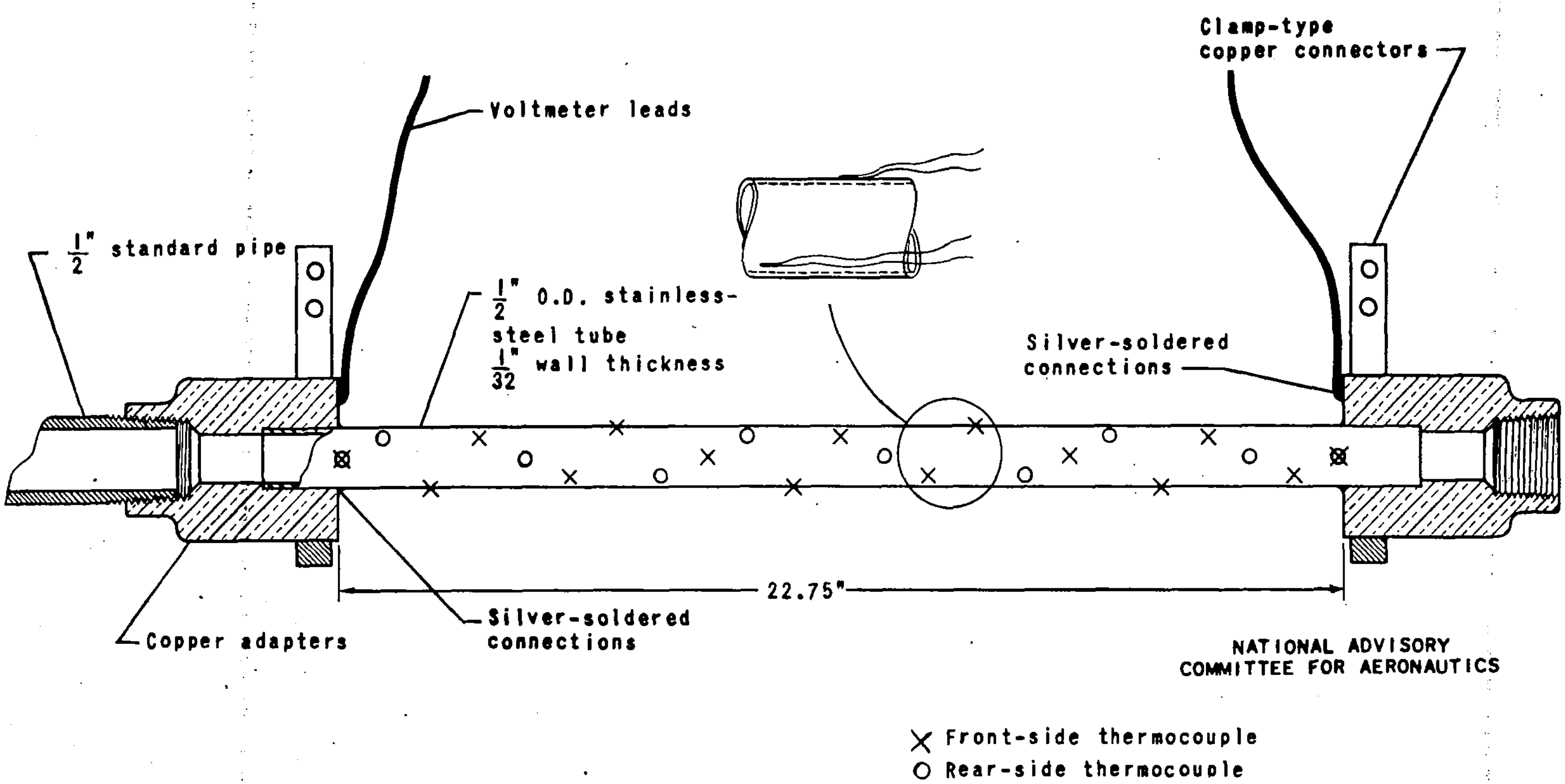


Figure 2. - Details of heater-tube section showing thermocouple locations and electrical-system and liquid-system connections.

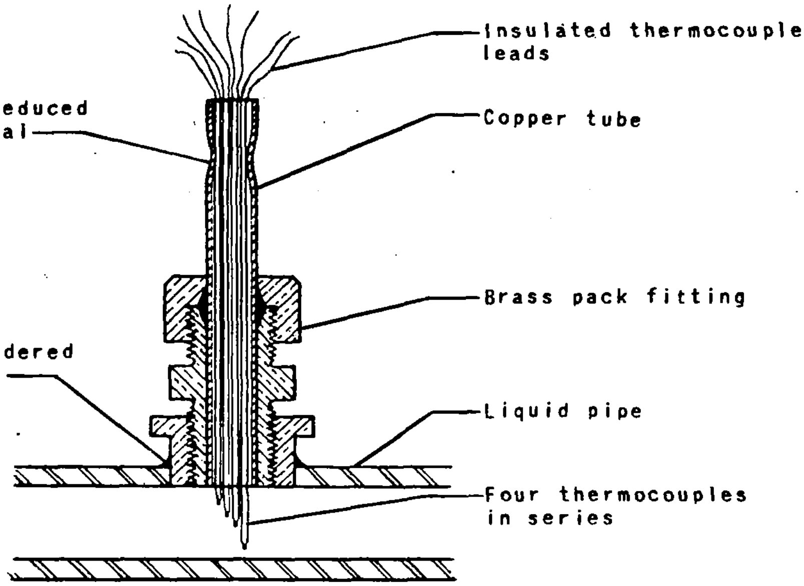


Figure 3. - Thermopile construction and installation in liquid pipes.

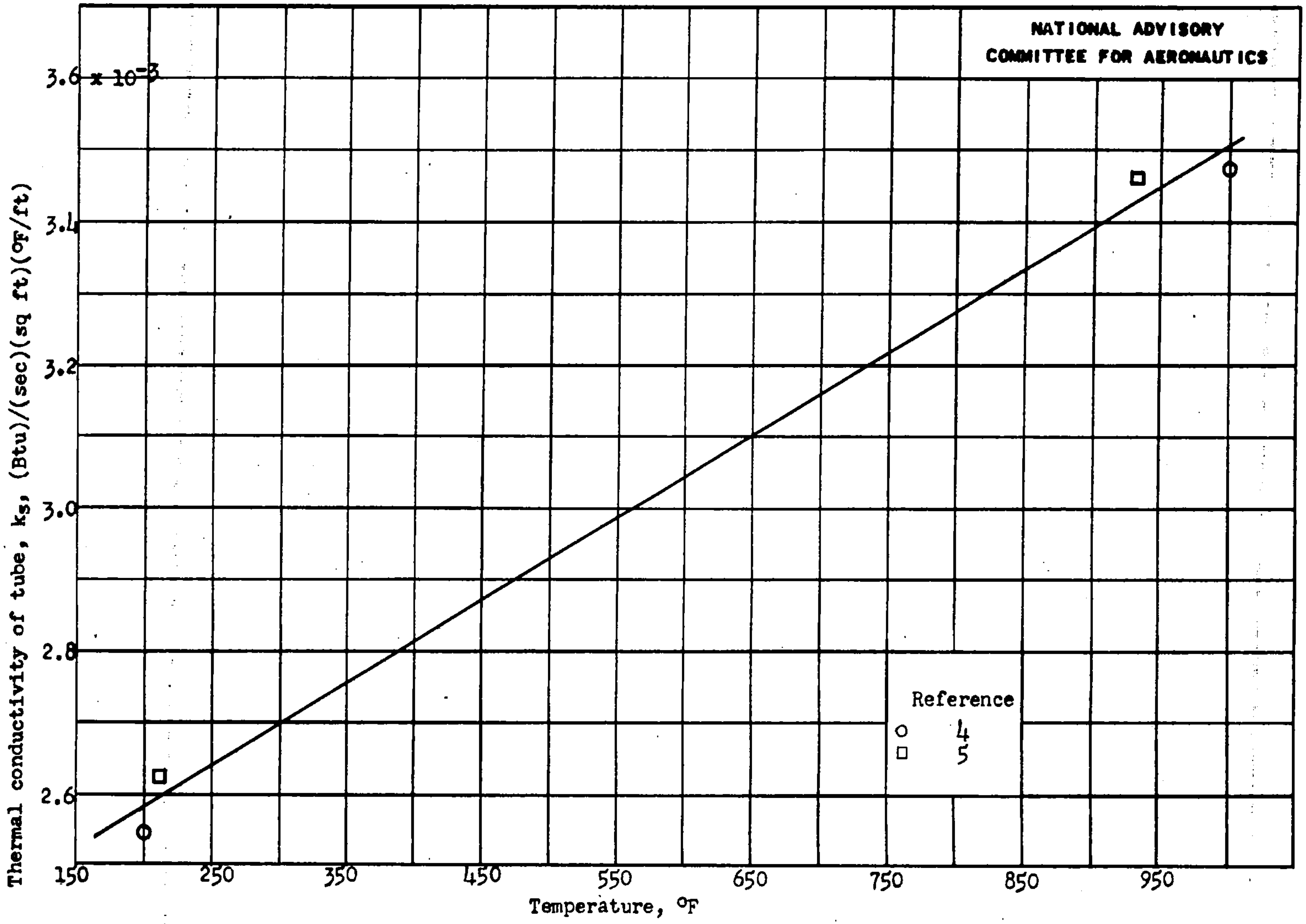


Figure 4.- Variation of thermal conductivity of 18:8 stainless steel with temperature. (Data from references 4 and 5.)

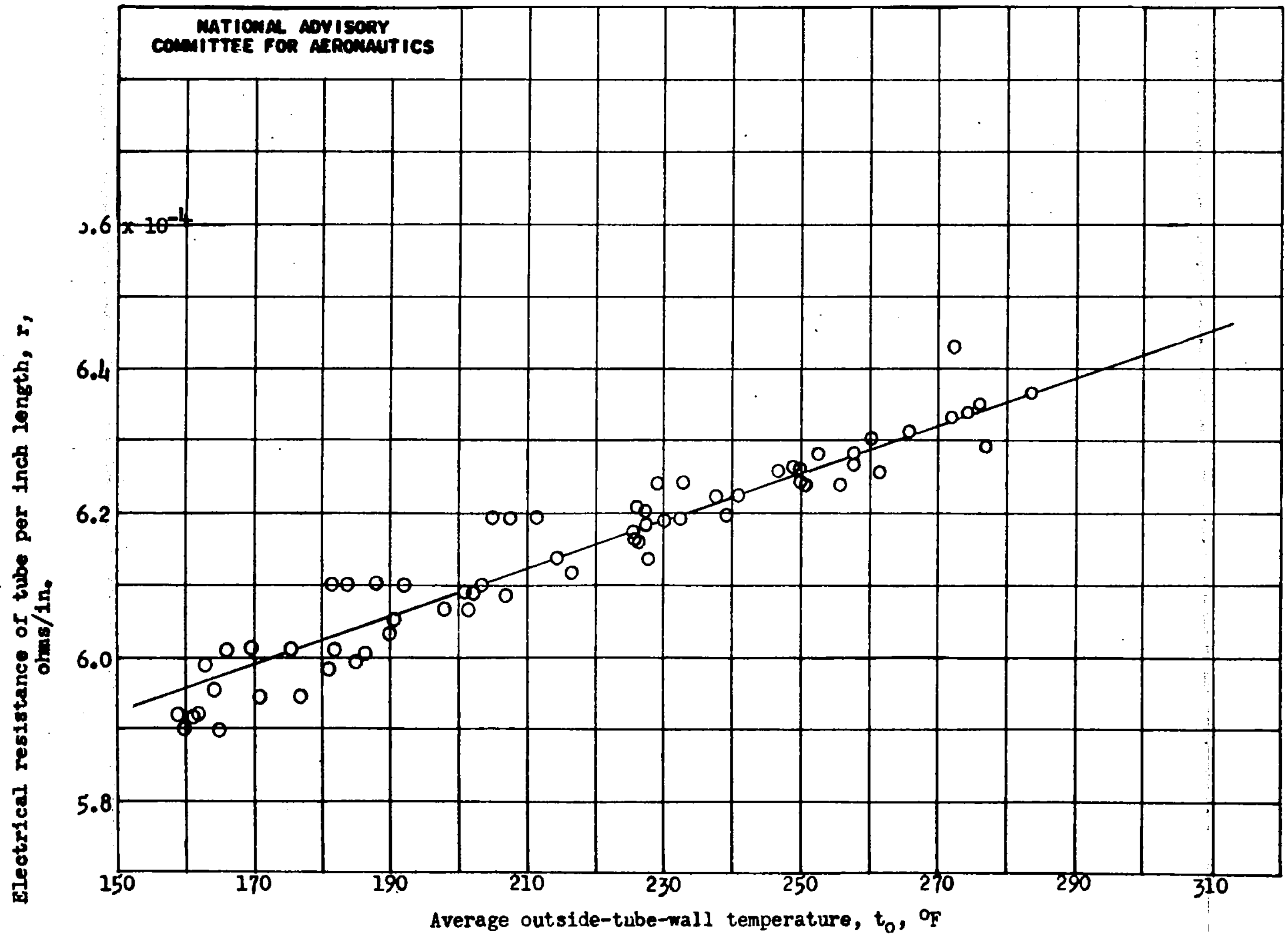


Figure 5.- Variation of electrical resistance of tube per inch length with average outside-tube-wall temperature as obtained from alternating-current measurements.

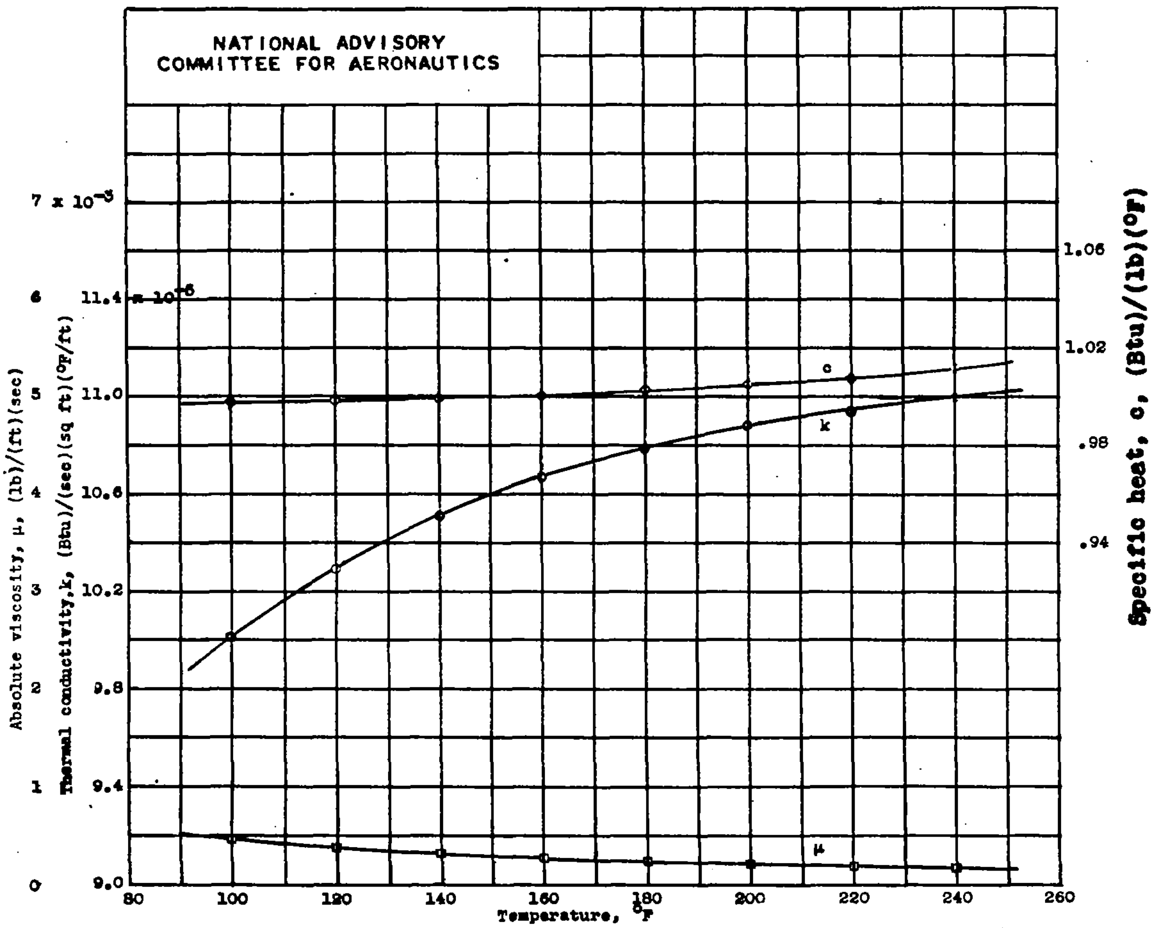
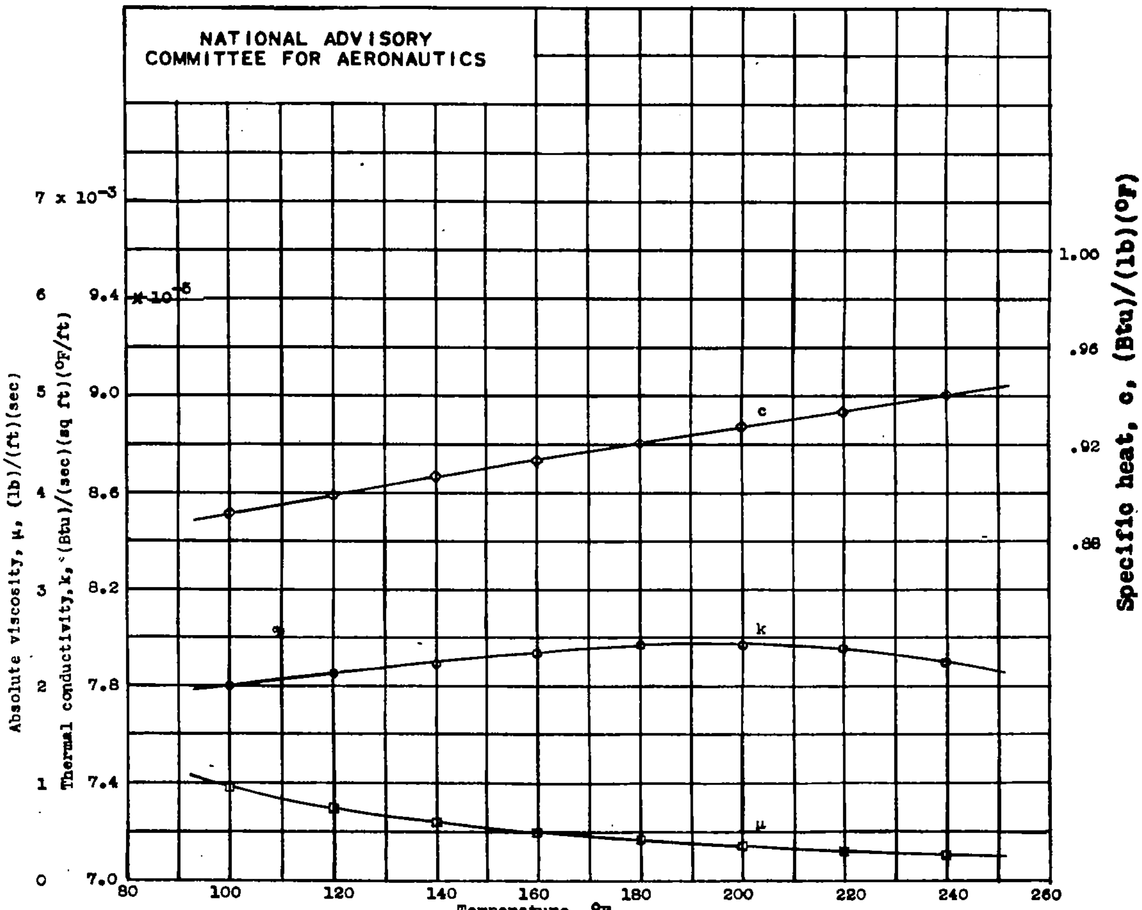
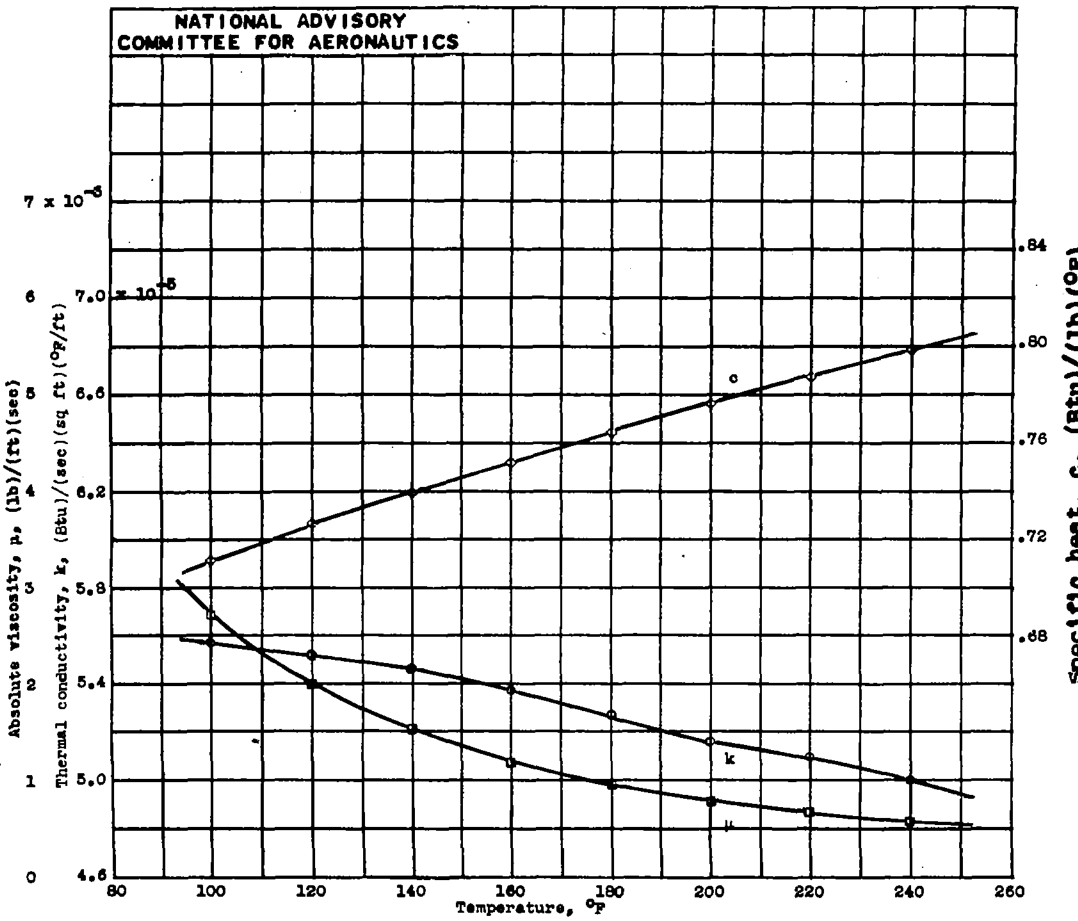


Figure 6. - Variation of specific heat c, thermal conductivity k, and absolute viscosity μ of water with temperature. (Data from reference 2.)



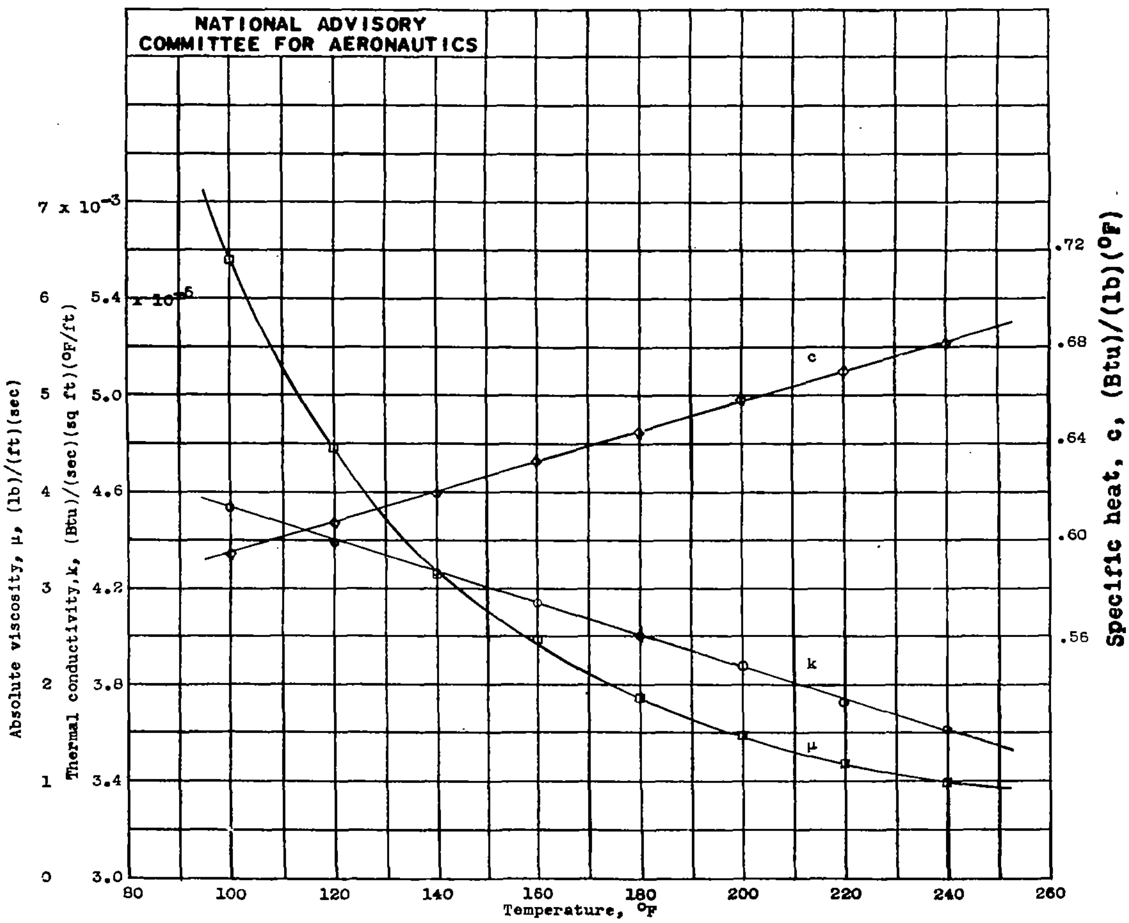
(a) Nominal (by volume) 30 percent-70 percent glycol-water solution.

Figure 7. - Variation of specific heat c, thermal conductivity k, and absolute viscosity μ of aqueous ethylene glycol solutions with temperature. (Data from reference 2.)



(b) Nominal (by volume) 70 percent-30 percent glycol-water solution.

Figure 7. - Continued.



(c) Nominal (by volume) 97 percent-3 percent glycol-water solution.

Figure 7. - Concluded.

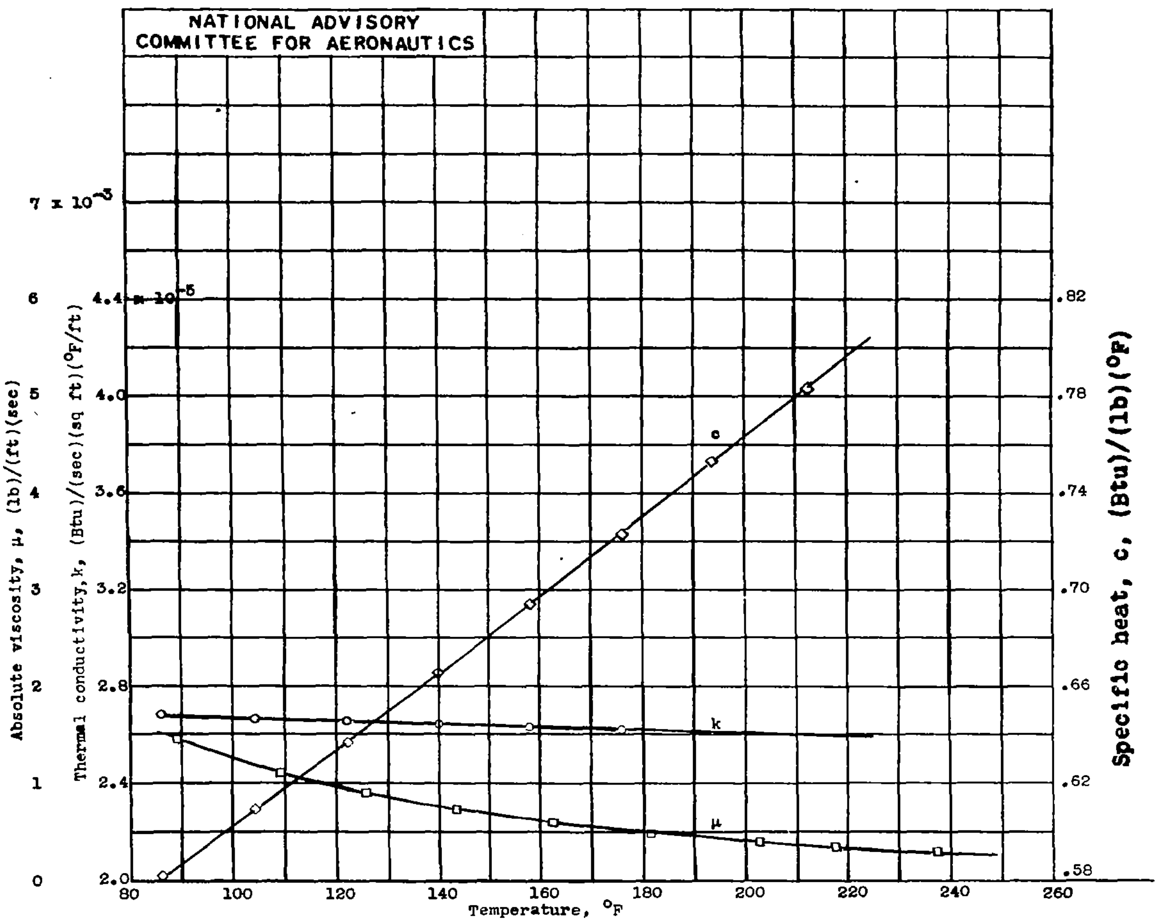


Figure 8. - Variation of specific heat c, thermal conductivity k, and absolute viscosity μ of butanol with temperature. (Data from references 6, 7, and 8.)

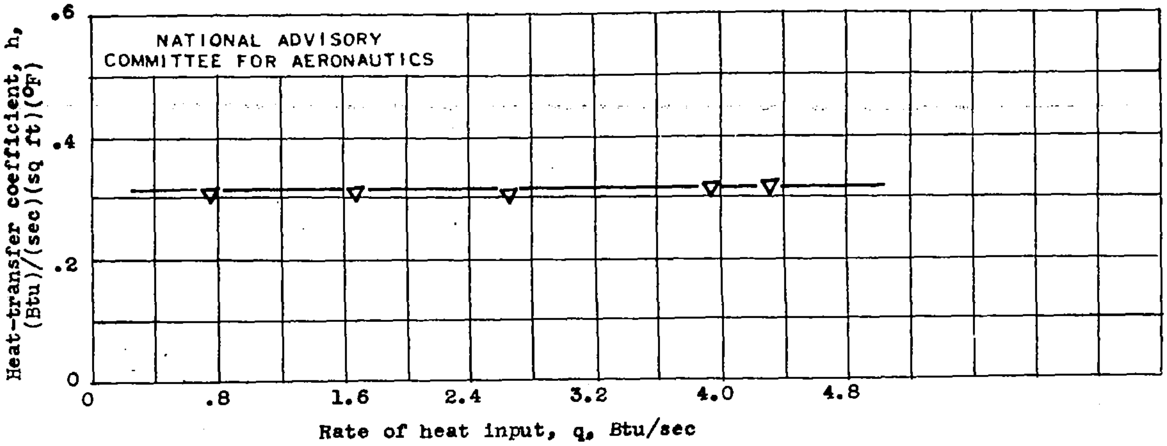


Figure 9.- Variation of heat-transfer coefficient with rate of heat input. Liquid, water; liquid-flow rate, 0.2 pound per second; average liquid temperature, 150° F; liquid pressure, 65 pounds per square inch absolute.

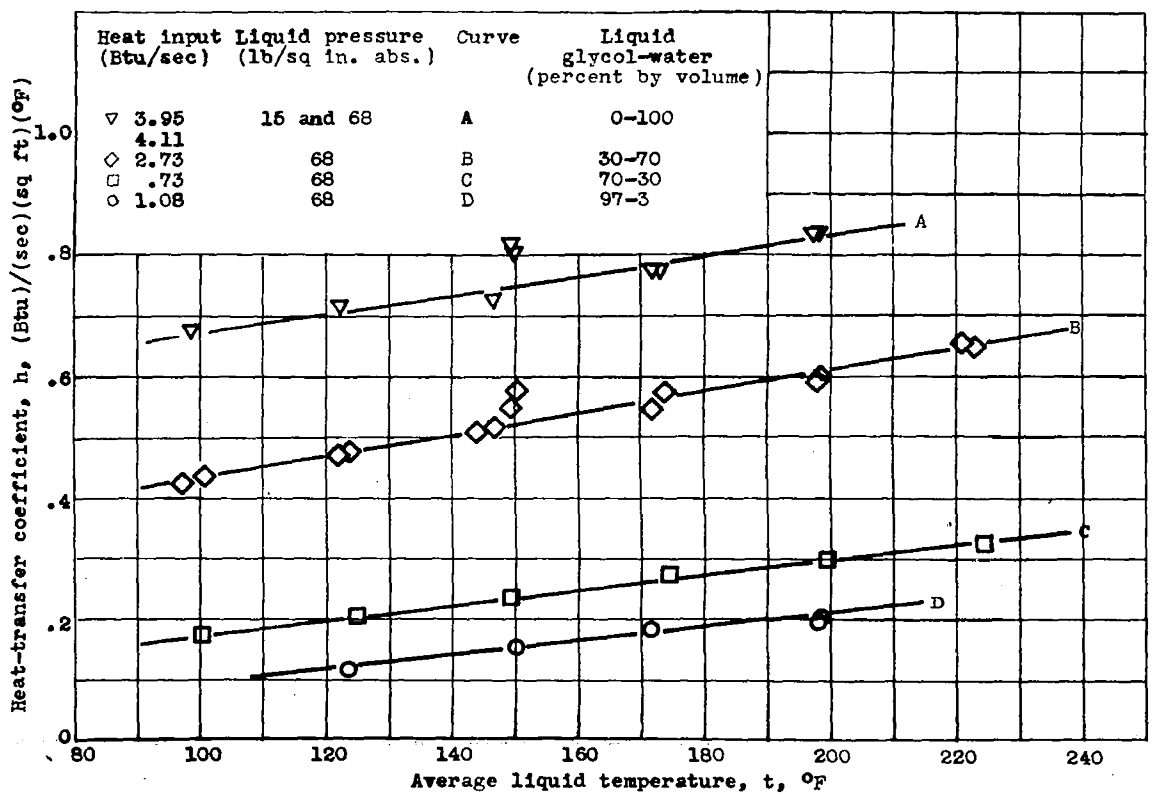
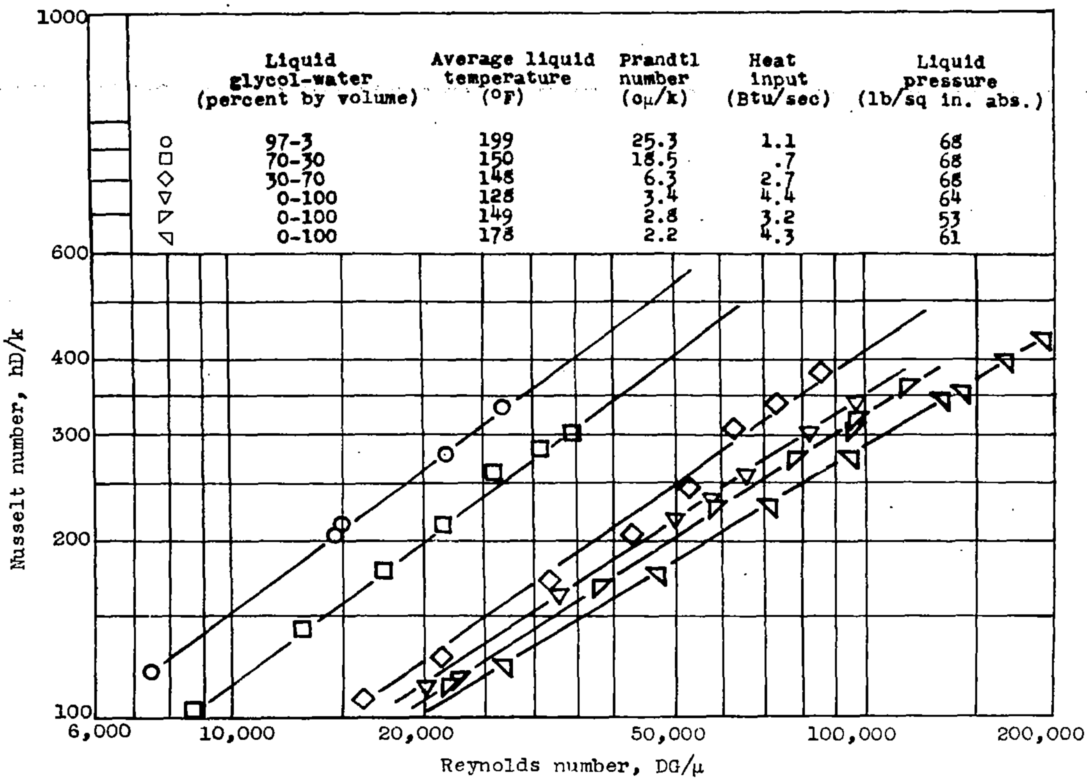
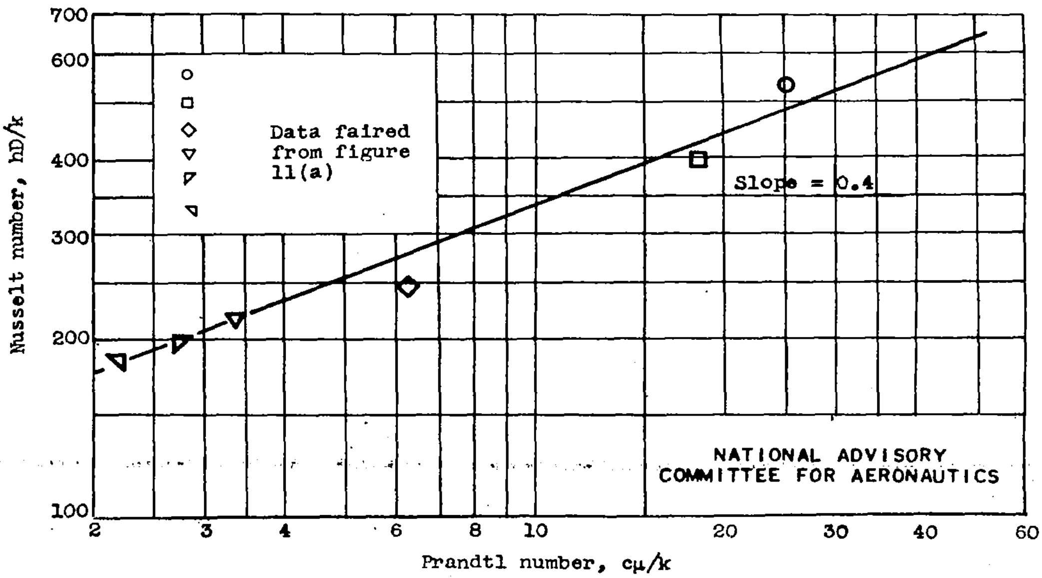


Figure 10.- Variation of heat-transfer coefficient with average liquid temperature for various glycol-water solutions at different approximately constant heat inputs. Liquid-flow rate, 0.67 pound per second.



(a) Variation of Nusselt number with Reynolds number for several liquids at different conditions of operation.



(b) Cross-plot of Nusselt number against Prandtl number at a Reynolds number of 50,000 for several liquids at different conditions of operation.

Figure 11.- Determination of exponent n on Prandtl number.

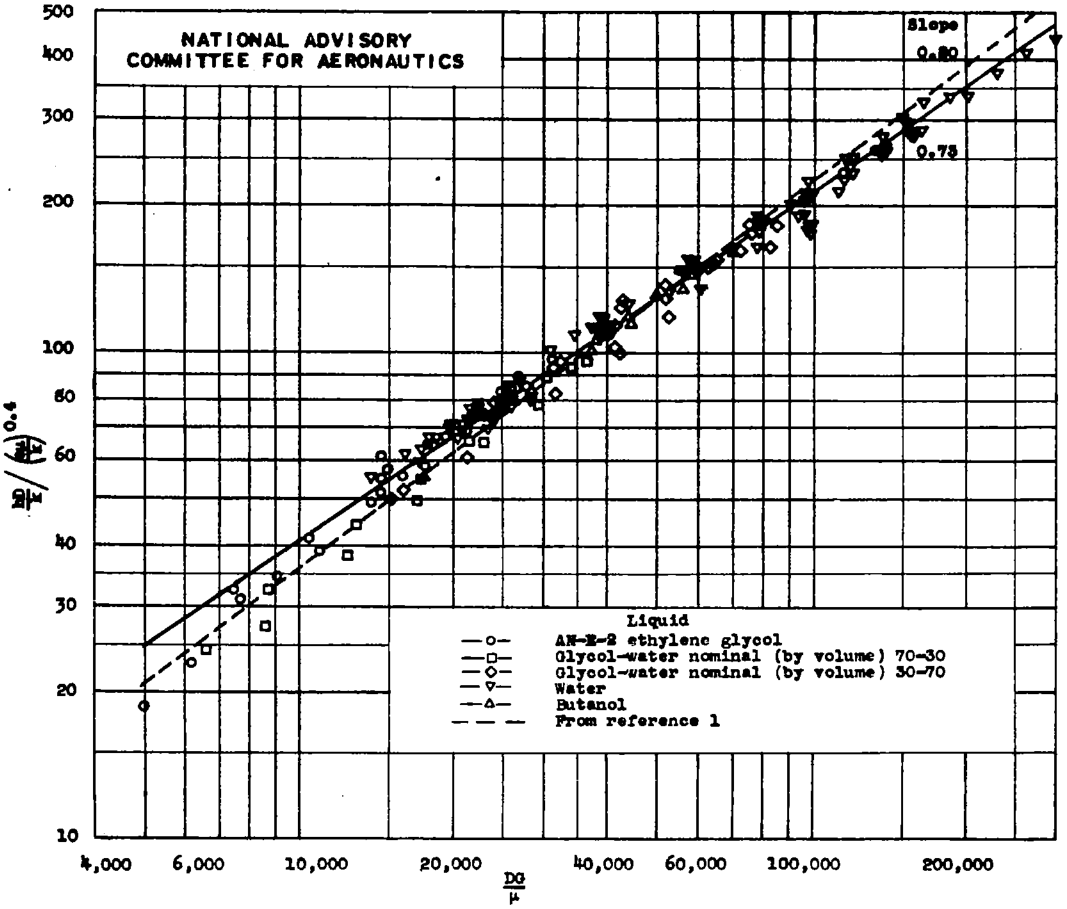


Figure 12.- Correlation of forced-convection heat-transfer data based on Nusselt number for several liquids flowing inside an electrically heated tube under various conditions of average liquid temperature, liquid-flow rate, liquid pressure, and heat input.

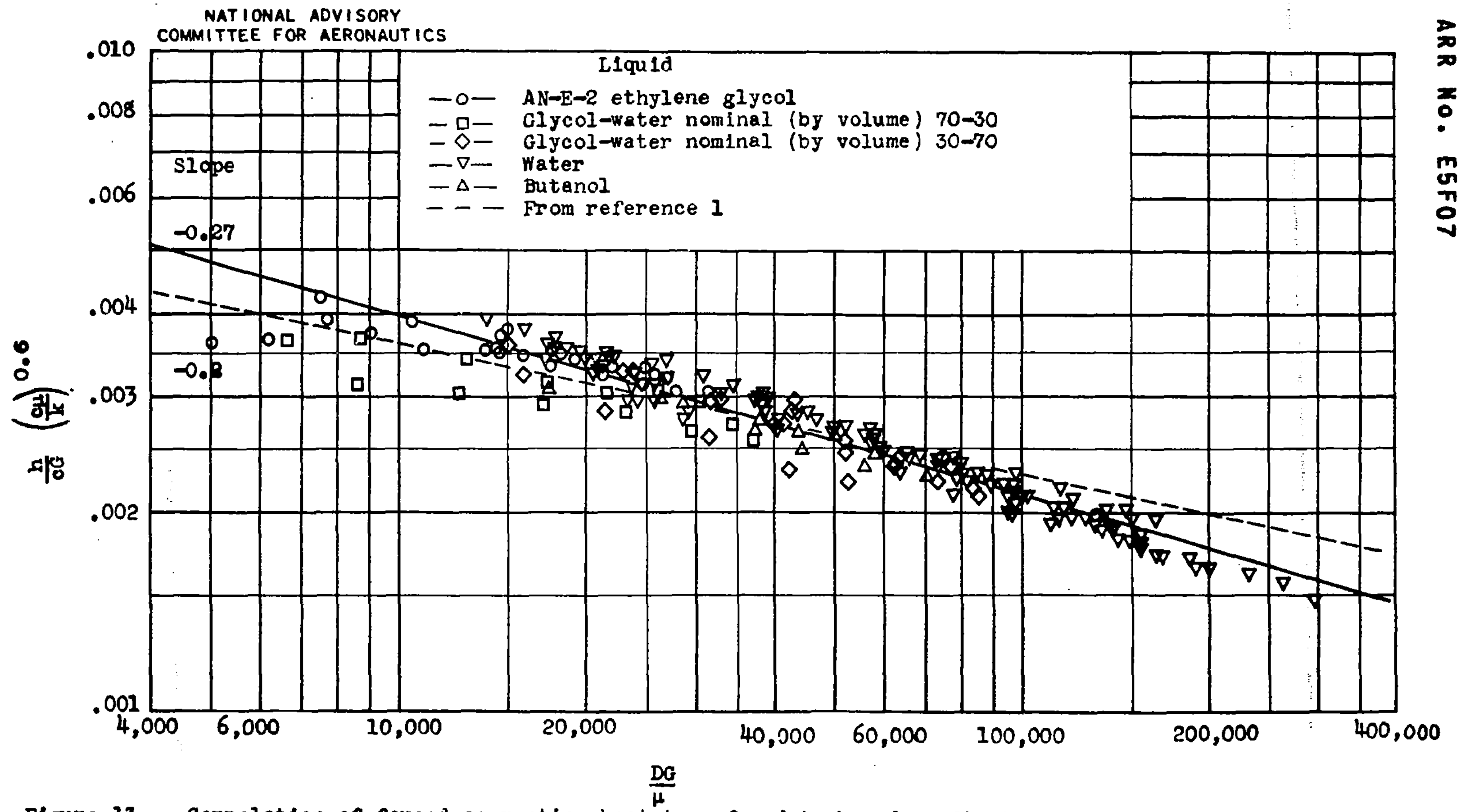


Figure 13.- Correlation of forced-convection heat-transfer data based on Stanton number for several liquids flowing inside an electrically heated tube under various conditions of average liquid temperature, liquid-flow rate, liquid pressure, and heat input.

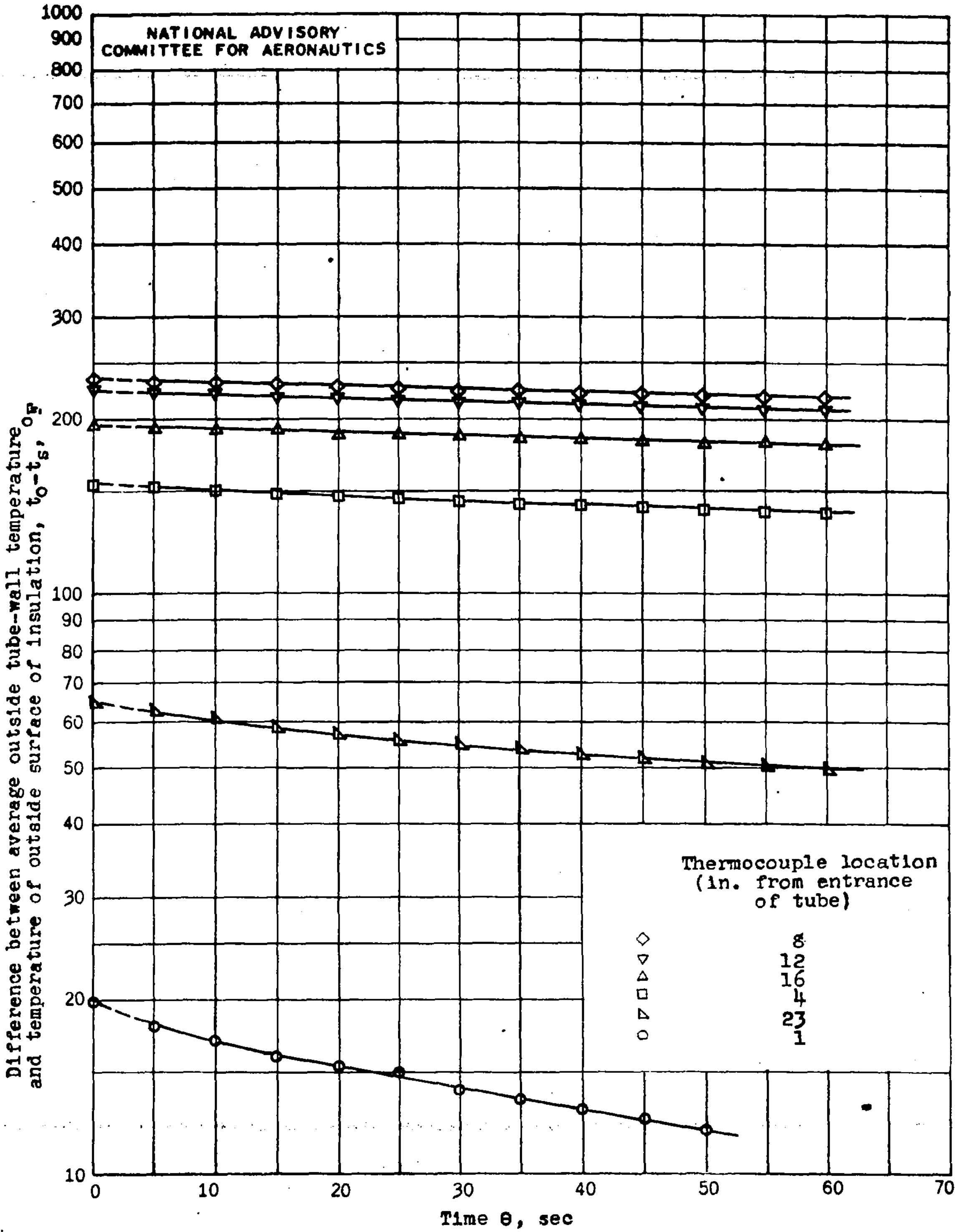


Figure 14.- Cooling-rate curves for several tube-wall thermocouples with no liquid in tube. Temperatures at zero time obtained with power being supplied to tube; temperature of insulation, 95°F.

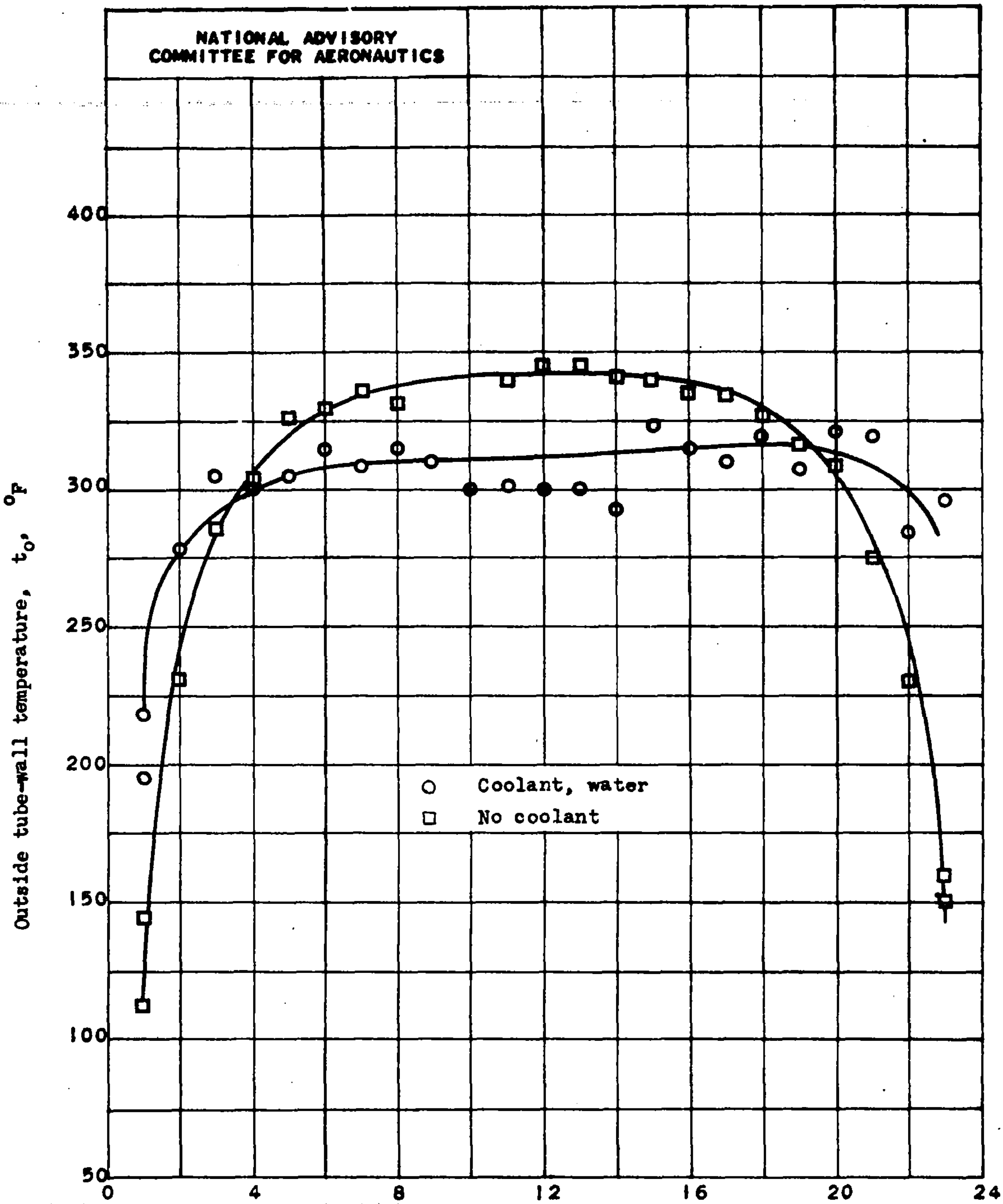


Figure 15.- Distribution of outside tube-wall temperatures under two different conditions of operation. With water cooling: average liquid temperature, 122° F; liquid-flow rate, 0.58 pound per second; power input, 1.9 Btu per second. When no coolant was used, power input was set equal to heat loss and all conditions were in equilibrium.